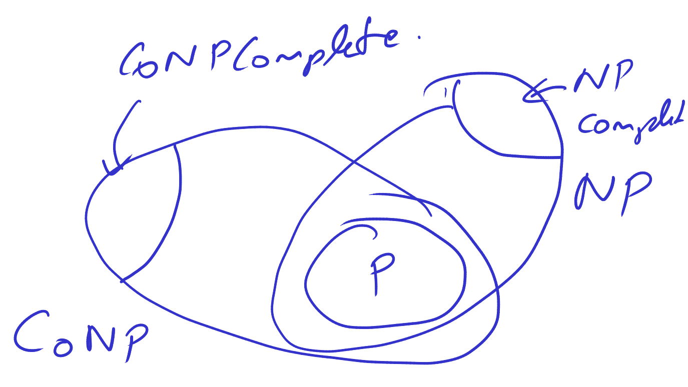
Previosly:

- $CoNP = \{L \mid \bar{L} \in NP\}$

- CoNP-Completeness.

$\bar{L} = \{0,1\}^* \setminus L$

$L \in NP\text{-Complete} \iff \bar{L} \in CoNP\text{-Complete}.$



- REACHABILITY $\in DSPACE(\log^2 n)$ ✓

$n^{\log n}$

Savitch's Theorem

$$NSPACE(f(n)) = DSPACE(f^2(n))$$

$NTIME(f(n)) \subseteq DTIME(2^{O(f(n))})$

Proof:

M is a $NSPACE(f(n))$ TM

How big is the conf's. graph of M?

$$m = 2^{f(n)}$$

$$\log^2 m = (\log 2^{f(n)})^2 = f^2(n)$$

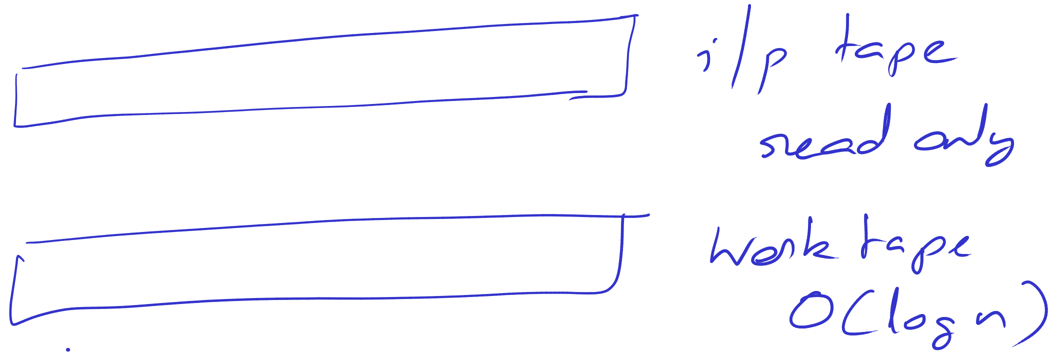
$\boxtimes$

$NPSPACE = PSPACE$

$NPSPACE = \bigcup_k NSPACE(n^k)$

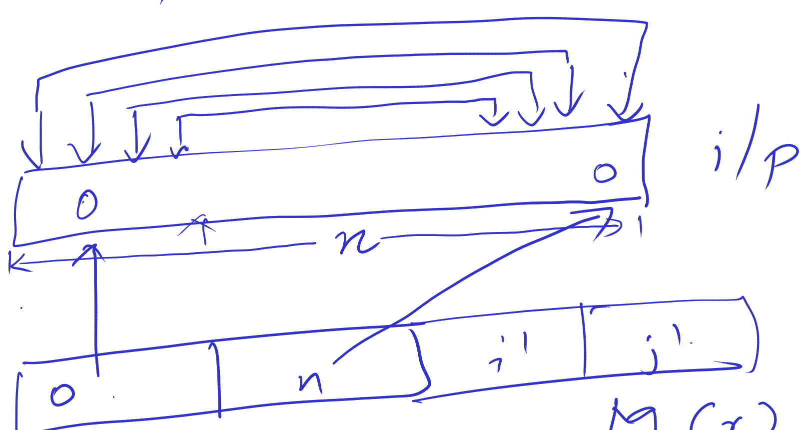
$PSPACE = \bigcup_k DSPACE(n^k)$

- $L = DSPACE(O(\log n))$



Reglar Languages $= DSPACE(O(1))$

PAL (palindromes) $\in L$



$M(x)$ {

$(i,j) \leftarrow \boxed{0 \mid n}$

while $(i \neq j)$

if $\boxed{x_i \neq x_j}$ return false

else $i = i+1,\ j = j-1$

}

Presentation Topic

$\mathrm{DSPACE}(o(\log\log n)) = $ Reg. Language

- $NL = \mathrm{NSPACE}(O(\log n))$

L vs NL?

$NL \subseteq \mathrm{DSPACE}(\log^2(n))$

Verifier definition of NL

$L \in NL$ if there is a TM $M(\cdot,\cdot)$

s.t. $x \in L \iff \exists c$ s.t. $M(x, c) = 1$

- M is a det. log-space TM
- c is on a seperate tape that whose heads stays at the same place or moves right
- $|c| \leq \mathrm{poly}(n)$

Reachability $\in$ NL

$\ni$

$\langle G, s, t\rangle$ iff the s-t path.

s-t-path

Verifier$(\langle G, s, t\rangle, c)$ {

// verify c is an s-t path in $\log n$ space.

$c = (v_1, v_2, v_3 \ldots v_n)$

for $i = 1$ to $n-1$

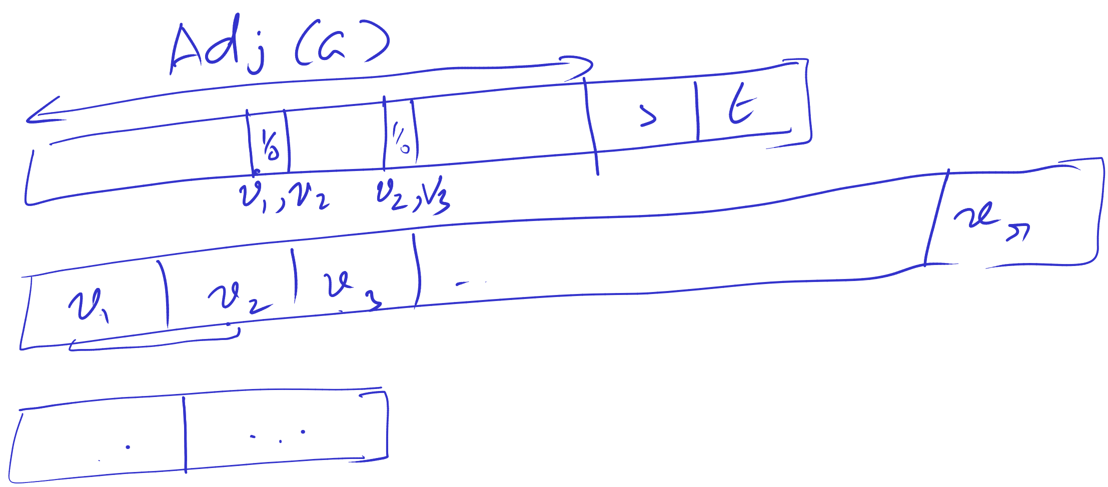
$(v_i, v_{i+1}) \in E(G)$

$v_1 = s$

$v_n = t$

}

$O(\log n)$



- $NL \subseteq DSPACE(\log^2 n)$

L vs NL

What is the hardest problem in NL?

Reductions.

NL-Complete

Can we use poly time reductions?

No. We want that if an NL-Complete has a $O(\log n)$ space algo then $NL = L$. If we use poly time reduction we cannot make that conclusion.

$NL \subseteq P$

Log-Space Reductions. $L \leq_L L'$

- $f : \{0,1\}^* \to \{0,1\}^*$
  s.t $x \in L \iff f(x) \in L'$

- There is a deterministic log space TM that given input (x, i) outputs the i^{th} bit of $f(x)$

Claim: $L_1 \leq_l L_2$ and $L_2 \leq_l L_3$ then $L_1 \leq_l L_3$ (transitivity.)

Proof:

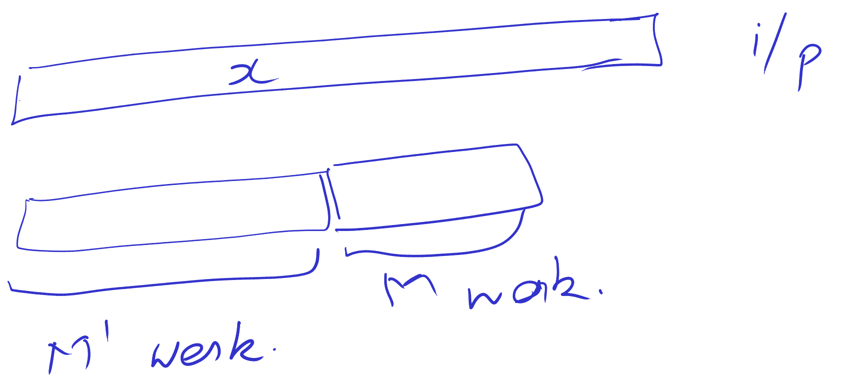
$L_1 \leq_l L_2$ $f(x)$ M

$L_2 \leq_l L_3$ $g(x)$ M'

$L_1 \le_\ell L_3$ $\quad g(f(x))$

We need TM on input (x, i),

compute i^{th} bit of $g(\underbrace{f(x)}_{i^{th}\text{ bit}})$



$x \xrightarrow{M} f(x)$

We cannot write down the full $f(x)$ ($O(n)$ bits)

Whenever M' requires a bit j of $f(x)$, M is run on $\underline{(x, i)}$ input to obtain it. We have to do this since we cannot store $f(x)$.

$\therefore L_1 \leq_l L_3$.

Suppose $L_3 \in L \Rightarrow L_1 \in L$?

Reachability $\in$ NL-Complete

(Analogue of Cook-Levin Theorem)

$L \in NL$

$L \leq_l$ Reachability.

x $\qquad \langle G, s, t \rangle$

Let M be the $NSPACE(O(\log n))$ TM for L. Configuration graph G of M is $\mathrm{poly}(n)$ in size. We need to check reachability between start and accept configuration in G.

$$\underbrace{(x, i)} \xrightarrow{M'} i^{th} \text{ bit in } \underbrace{\langle G, c_{start}, c_{acc} \rangle}$$

$$(x, e) \xrightarrow{M''} (e \in E(G))$$

If Reachability $\in L$ then $L = NL$.

Presentation Topic

Undirected Reachability $\in L$

- $CoP = \{L \mid \bar{L} \in P\}$ vs P

  $CoP = P$?

- $CoL = L$

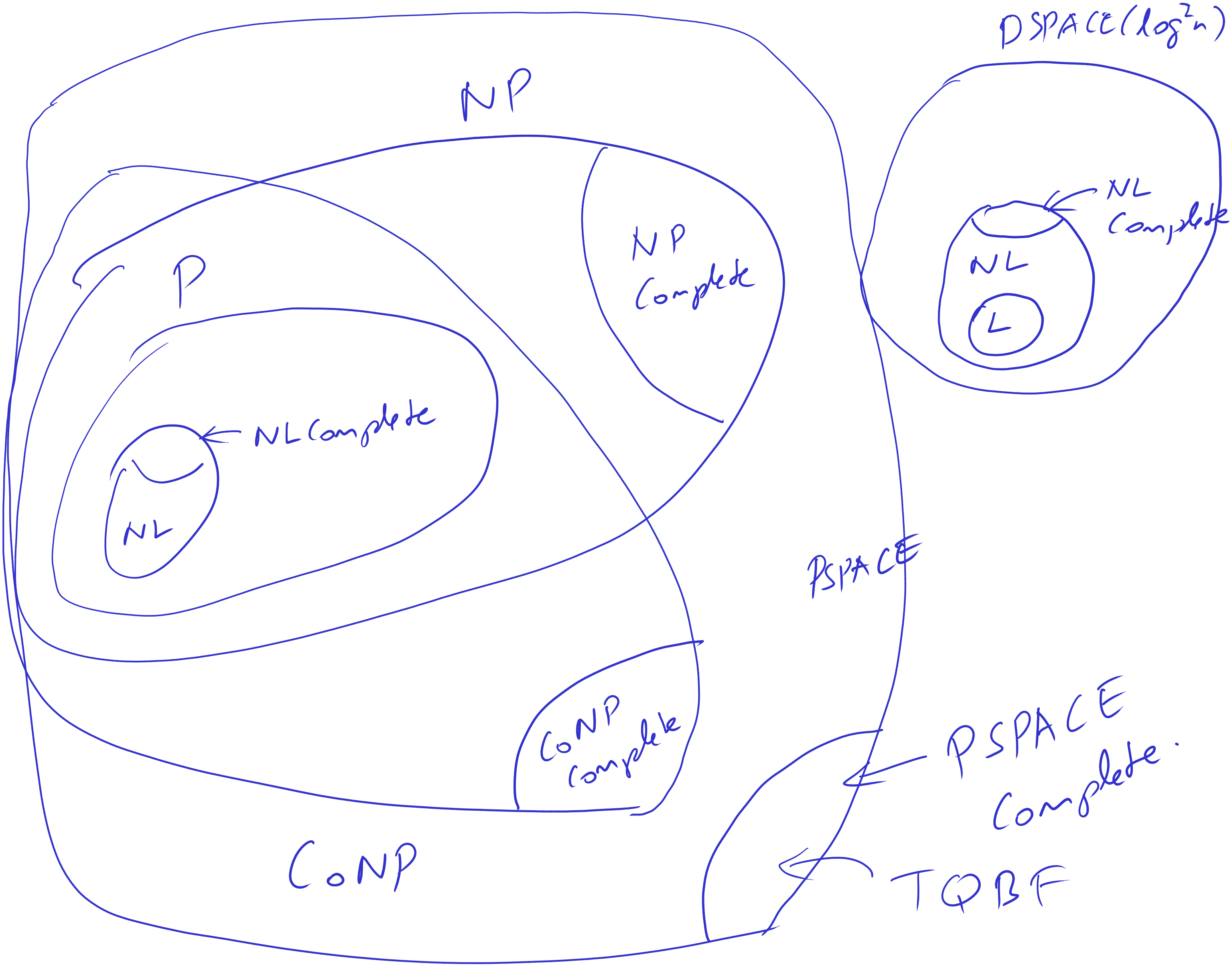
- $CoNL$ vs NL

- $CoPSPACE$ vs $PSPACE$?

Presentation Topic

$CoNL = NL$

(Immerman - Szelepcsényi Theorem)



Presentation Topics.

PSPACE Completes to Complexity of Game Players.

It is still possible for $\underline{L = NP}$.

$L \subsetneq PSPACE$