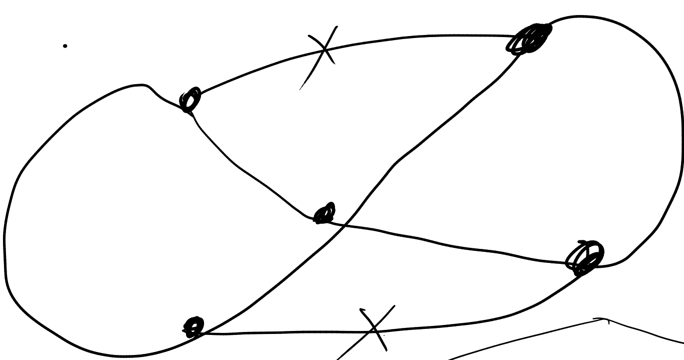
HAM-CYCLE

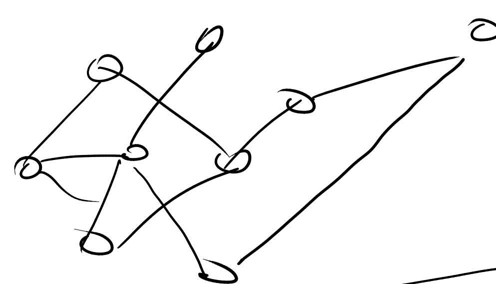
Input: G(V, E)

a cycle which passes through every vertex exactly once.

Output:

- Descison : YES/NO 1/0
- Search : A Hamiltonian cycle in G.
- Counting : Find the no. of hamiltonian cycles.





|E| steps Search problem is solved provided Descison problem can be solved.

BOOL-FORMULAE

Input: $\underbrace{(x_1 \wedge x_2) \vee \neg x_3 \vee (\neg x_4 \wedge x_1)}_{\varphi}$

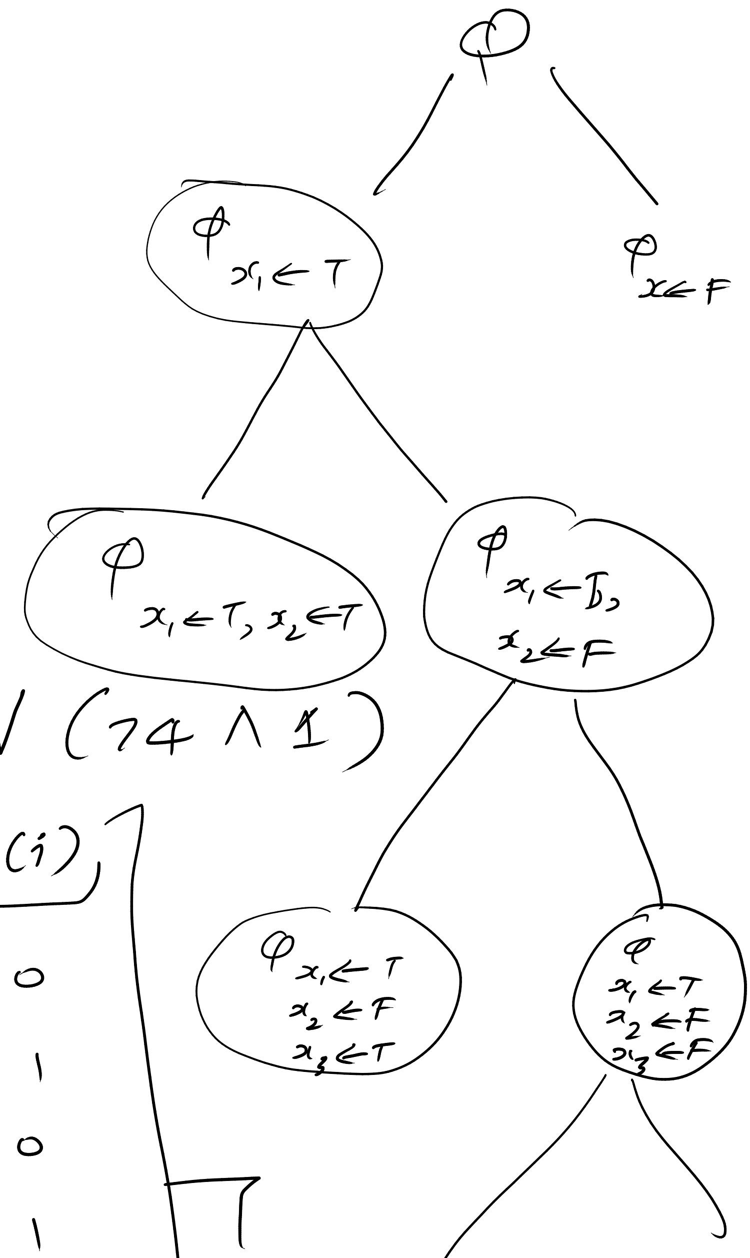
Output:

- Descison: Is there an an assignment for $x_1 \ldots x_n$ that makes φ true.
- Search: Find the assignment that makes φ true
- Counting: Count # of assignments that make φ true.

$$
\begin{tikzcd}
 & \varphi \arrow[dl] \arrow[dr] & \\
\varphi[x_1 \leftarrow F] & & \varphi[x_1 \leftarrow T]
\end{tikzcd}
$$

$t(n)$

$2n\,t(n)$



$(1 \wedge 2) \vee \neg 3 \vee (\neg 4 \wedge 1)$

$i \longmapsto 1\,\underbrace{\text{bin}(i)}$

$\wedge \longmapsto 0000$

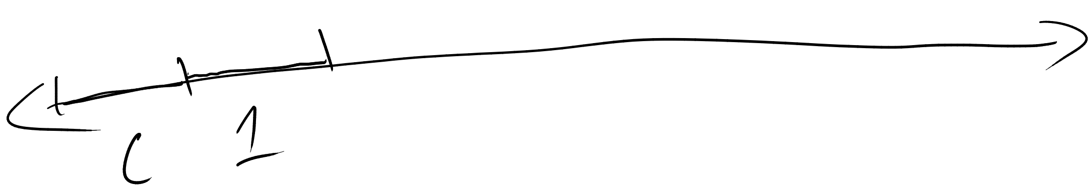
$\vee \longmapsto 0001$

$\neg \longmapsto 0010$

$(\longmapsto 0011$

$) \longmapsto 0100$

Prefix free encoding.



Input / Outputs encoded in Binary.

- Decision   output 0/1

0 1 1 0 | 1 0 1 1 | 0 1 1 1 $\in \{0,1,|\}^n$

$0 \to 00$
$1 \to 01$
$| \to 1$

"Huffman Encoding"
"Prefix free"

$\{0,1\}^{2n}$

00 11 11 00 01 11 00 11 11 01 00 11 11 11

Boolean Fn rep of Comp. Problem.

$$f : \{0,1\}^* \to \{0,1\}$$

$f(x) = 1$ iff on input x output $= 1$

$f(\langle G \rangle_{0,1}) = 1$

iff G has a Ham. cycle.

$f(\langle \varphi \rangle_{0,1}) = 1$ iff

φ has a solution

Language Rep.

$L_{HAM\text{-}CYCLE} \subseteq \{0,1\}^*$

$x \in L_{HAM\text{-}CYCLE}$ iff $dec(x) = G(V,E)$ has a ham cycle.

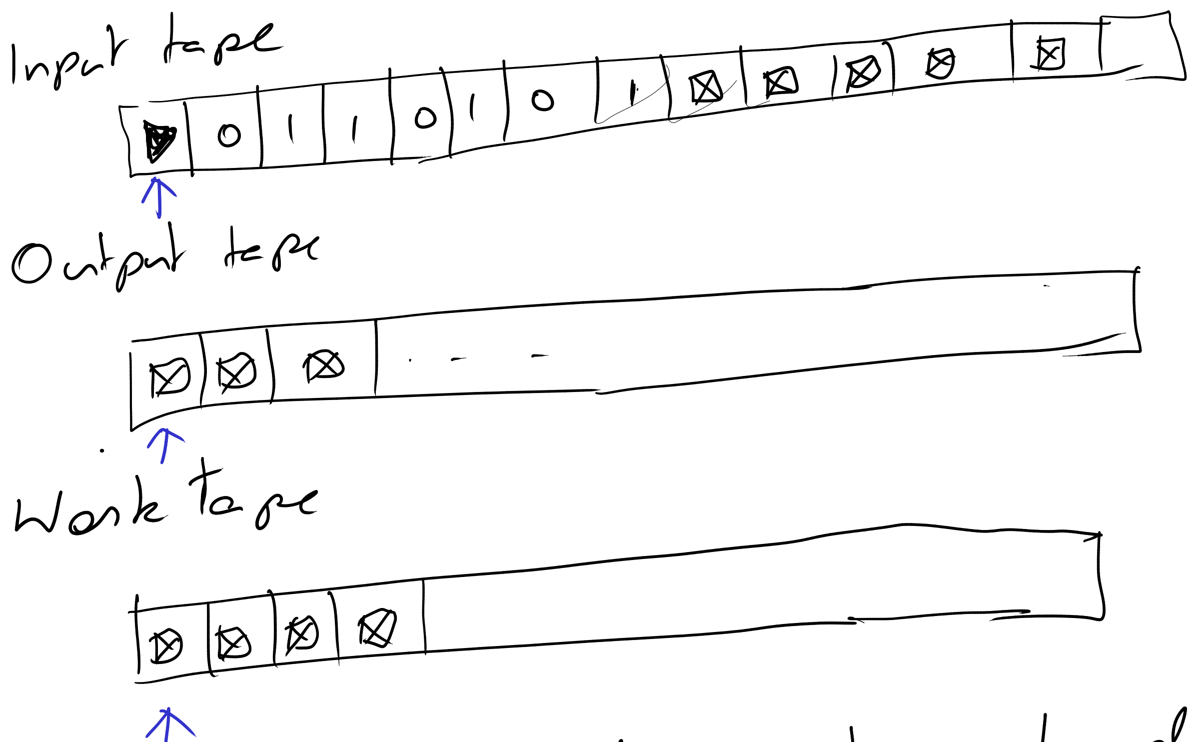
$f^{-1}(1) \subseteq \{0,1\}^*$

Church : λ-calculus.

Turing : <u>Turing Machine</u>.

set of tape symbols

$\Gamma = \{0, 1, \triangleright, \boxtimes\}$



Based on symbols on tape heads

- decide to overwrite worktape or output tape symbol.
- move head Left/Right in any of the tapes

- remember finite (constant) in memory.

Q : state space (will not grow with input size)

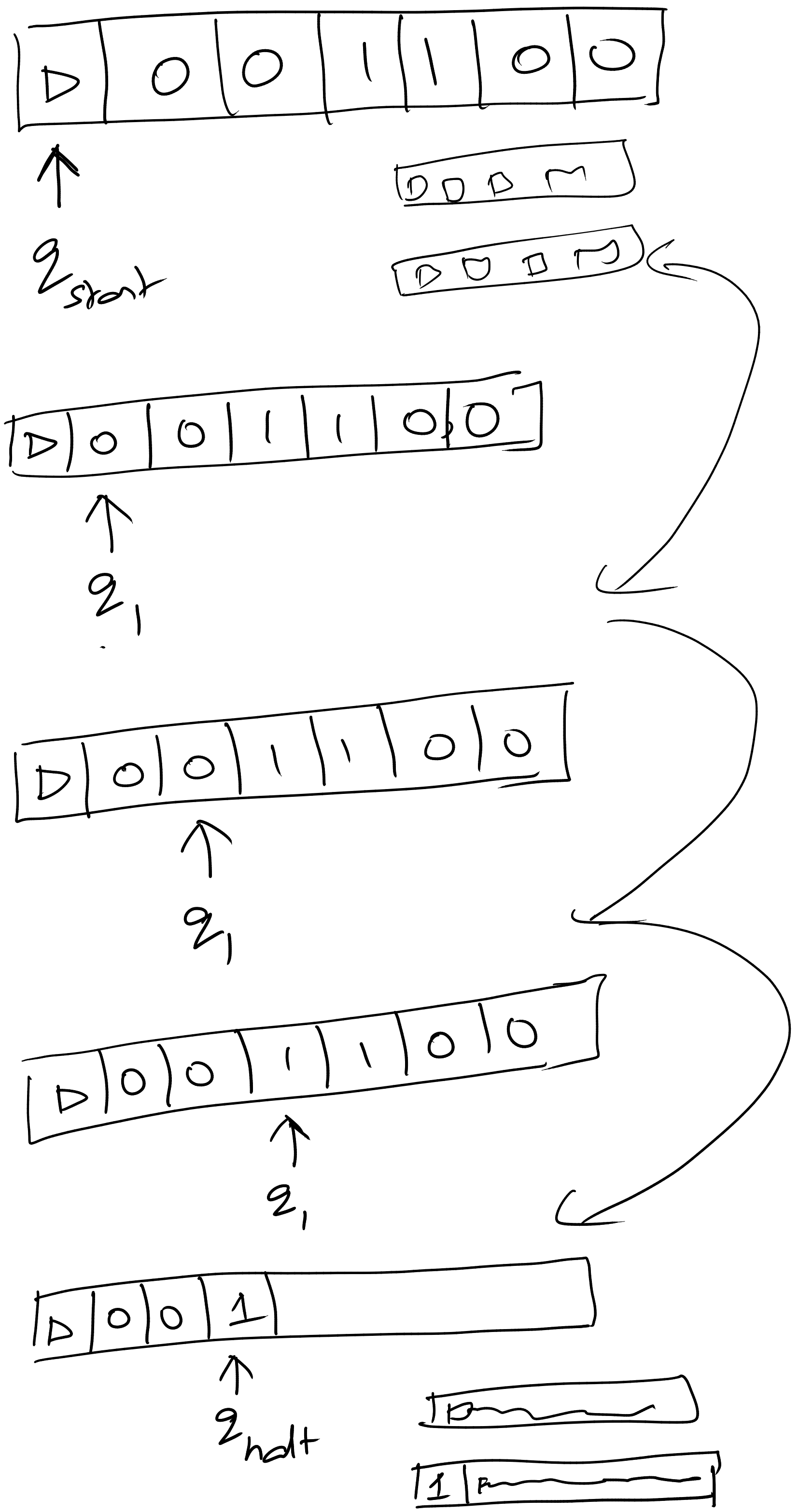
$$\boxed{\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}}$$

Example: $Q = \{q_{start}, q_1, q_{halt}\}$ ✓

$\Gamma = \{0, 1, \triangleright, \square\}$ ✓

δ

Q	IT	Q	OT	{L, R, S}
q_{start}	$\triangleright$	q_1	–	R
q_1	0	q_1	–	R
q_1	1	q_{halt}	1	S
q_1	$\square$	q_{halt}	0	S


▷ 0 0 1 1 0 0
q_{start}
q_1
q_1
q_1
▷ 0 0 1
q_{halt}
1

$$TM = \boxed{(\Gamma, Q, \delta)}$$

numbers

$$bin(|\Gamma|), |Q|, (q_s, D, q_1, -, R),$$

$$(\qquad)$$

$$2 + 1 + 2 + |Q| + |\Gamma| + 2 = O(|Q| + |\Gamma|)$$