Previosly:

- Decidable.
- "Halting Problem"
- Rec, RE languages.
- Universal Turing Machine ("interpreter")
  - input: $\langle M, x \rangle$
  - output: $M(x)$
- Robustness:

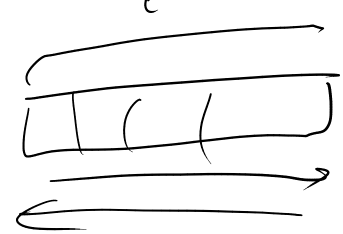
3 tape ← 10 tape

binary ← 1000 alphabets.

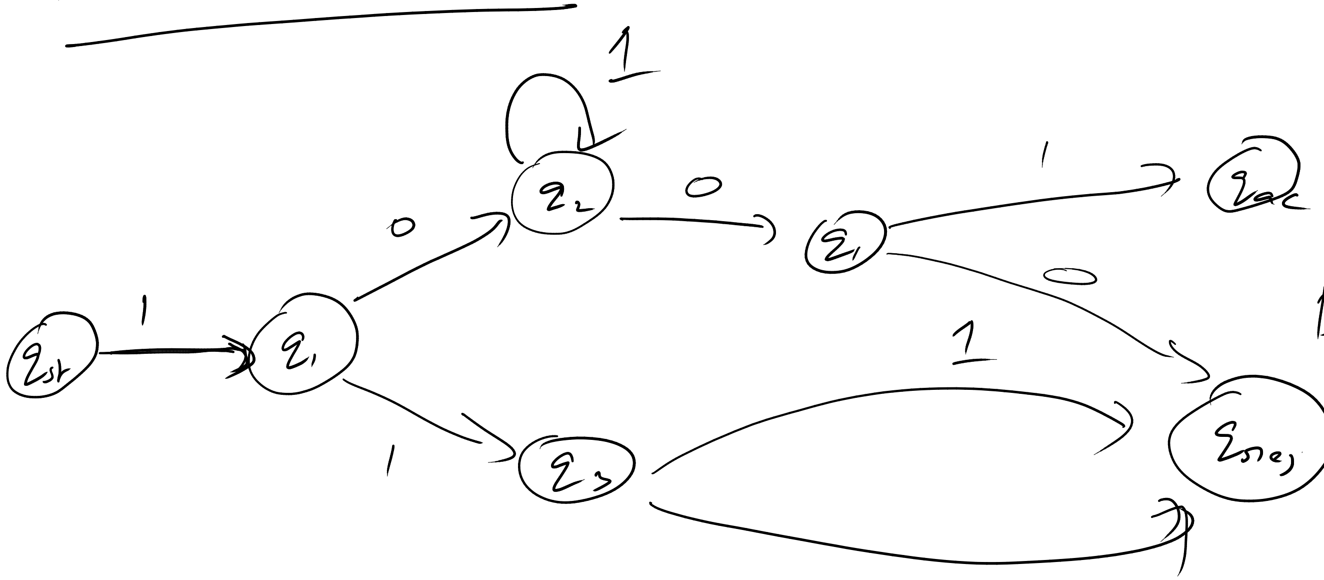
2 tape ← k tape

kt^2 t

t

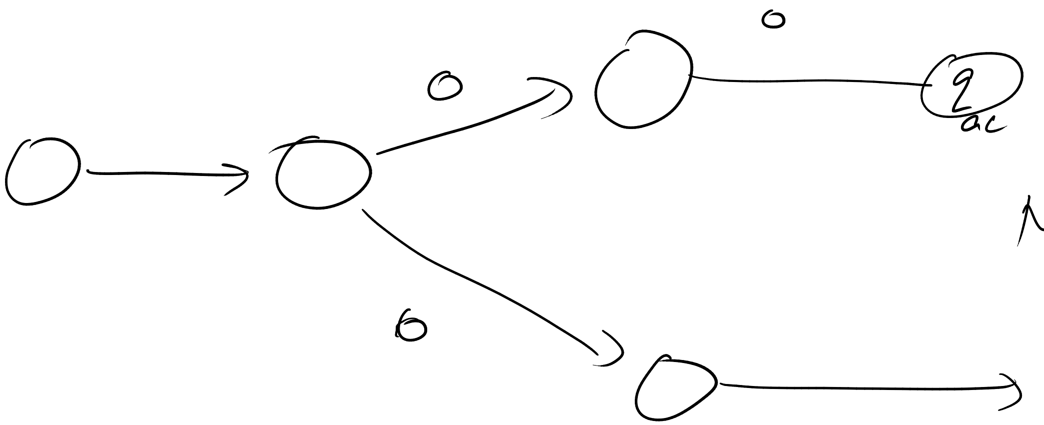


Nondeterminism



Det

DFA



NFA

Non det

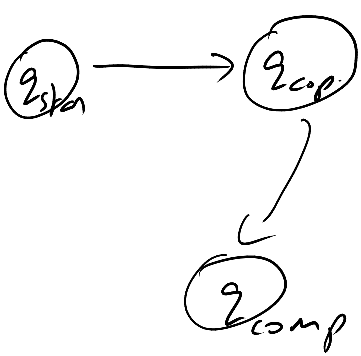
Set of languages decided by det finite automata

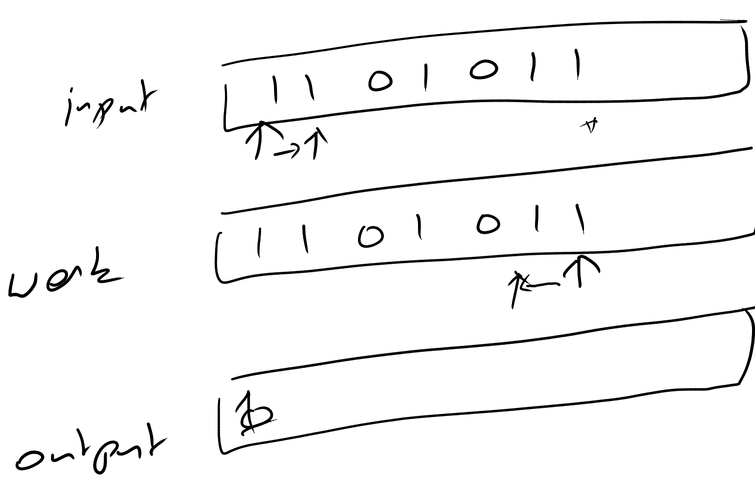
$\equiv$ Regular Languages.

L is decided by Nondeterministic finite Automata

Can TM solve more problems than DFA?

$$L_{PAL} = \{ x \in \{0,1\}^n : x = Rev(x) \} \in Rec$$





$$q_{st}, \overset{ip}{\triangleright} \longrightarrow q_{copy}, R$$

$$q_{copy}, \overset{ip}{0/1} \longrightarrow \overset{output}{o/p}$$

$L_{PAL} \notin$ Reg Languages. $\subsetneq$ CFL

CSL

~~"Pumping Lemma"~~

Languages dec by DFA = Languages dec by NFA

$\delta : Q \times \Sigma \rightarrow Q$

$Q \times \Sigma \times Q$

"power set construction"

For every NFA, construct DFA whose states are powerset of states of NFA.

Nondeterministic Turing Machine (NTM)

Δ fn for NFA $\subseteq Q \times \Sigma \times Q$

$\subseteq Q \times \Gamma^3 \longrightarrow Q \times \Gamma^3 \times \{L, R\}^3$

Languages decided by NTM $\supseteq$ $=$ Languages Decided by DTM

n^2 $\qquad 2^{O(n^2)}$

Configuration of a TM

(q, position of tape heads, ___ , ___ , ___)

C_i $\qquad \{1, \ldots t\}^3$

Suppose TM M decides L in f(n) steps

$\forall x \in \{0,1\}^n$,

M runs for $\leq f(n)$ steps and decides x

ie $M(x) = 1 \iff x \in L$

- M in NTM

- Can we design a Det Turing Machine for L?

- On input $x \in \{0,1\}^n$,

  there exist a path for M

  that accepts using n^2 steps.

Configuration Graph

$$C_{start} = \left(q_{start},\ 0,0,0,\ \boxed{\quad x \quad},\ \boxed{\sqcup\sqcup\sqcup\ \ \sqcup\sqcup},\ \underbrace{\boxed{\sqcup\sqcup\sqcup\sqcup\sqcup\sqcup\sqcup}}_{n^2}\right)$$

bit size of configuration

$$= \cancel{\log Q} + 3\log n^2 + 3n^2$$

$$= O(n^2)$$

How many configurations are possible?

$$2^{O(n^2)}$$

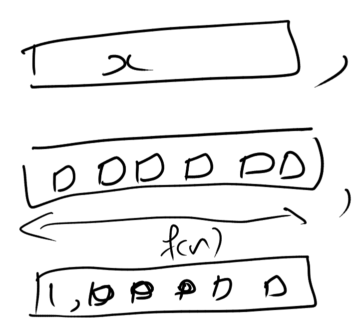
$V =$ all configs $\qquad |V| = 2^{O(n^2)}$

$E = \{ (c, c') :$ if c' can be obtained from c using delta fn $\}$

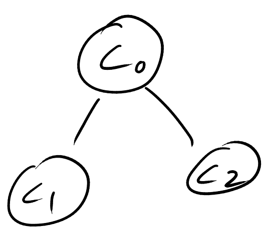
$G = (V, E)$

In G, is there a path of length n^2 from c_{start} to

$c_{acc} = (q_{acc}, 0, 0, 0,$



$)$?



Can we solve using DTM?

$2^{O(f(n))}$

$D \subseteq Q \times \Gamma^3 \times Q \times \Gamma^3 \times \{L, R\}^3$

$M'(\underline{x})$ {

Write all vertices in V for loop over $2^{O(n^2)}$

Write all edges in E

for $v \in 2^{O(n^2)}$

for $w \in 2^{O(n^2)}$:

if (v, w) is valid by D,

then $(v, w) \in E$

else $(v, w) \notin E$

$2^{2O(n^2)}$

Check if C_{acc} is reachable

from C_{start}. in n^2 steps.

}

Languages decided by

NTM = DTM.

DTIME(f(n))

= set of all languages decided by a DTM in time $\leq f(n)$

NTIME(f(n)) = " NTM in time $\leq f(n)$

$$\boxed{\begin{array}{c} DTIME(f(n)) \subseteq NTIME(f(n)) \\ \cap \\ DTIME\left(2^{O(f(n))}\right) \end{array}}$$

DSPACE $(f(n))$

= set of all languages decided by a DTM with space $\leq f(n)$

NSPACE $(f(n))$ = " NTM with space $\leq f(n)$

$$\mathrm{DTIME}(f(n)) \subseteq \mathrm{DSPACE}(f(n)) \subseteq \mathrm{DTIME}\left(2^{O(f(n))}\right)$$

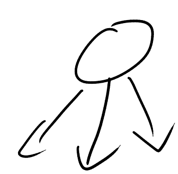
$$\mathrm{NTIME}(f(n)) \subseteq \mathrm{NSPACE}(f(n))$$

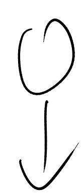
$$\boxed{\mathrm{NSPACE}(f(n)) = \mathrm{DSPACE}((f(n))^2)}$$

↳ Savitch's Thm (next lecture)

$$\mathrm{DTIME}(f(n)) \subseteq \mathrm{NTIME}(f(n)) \subseteq \mathrm{DTIME}(2^{O(f(n))})$$

$$\mathrm{DTIME}(f(n)) \subseteq \mathrm{DSPACE}(f(n)) \subseteq \mathrm{DTIME}(2^{O(f(n))})$$





$$\boxed{P = \bigcup_{k \in \mathbb{N}} DTIME(n^k)}$$

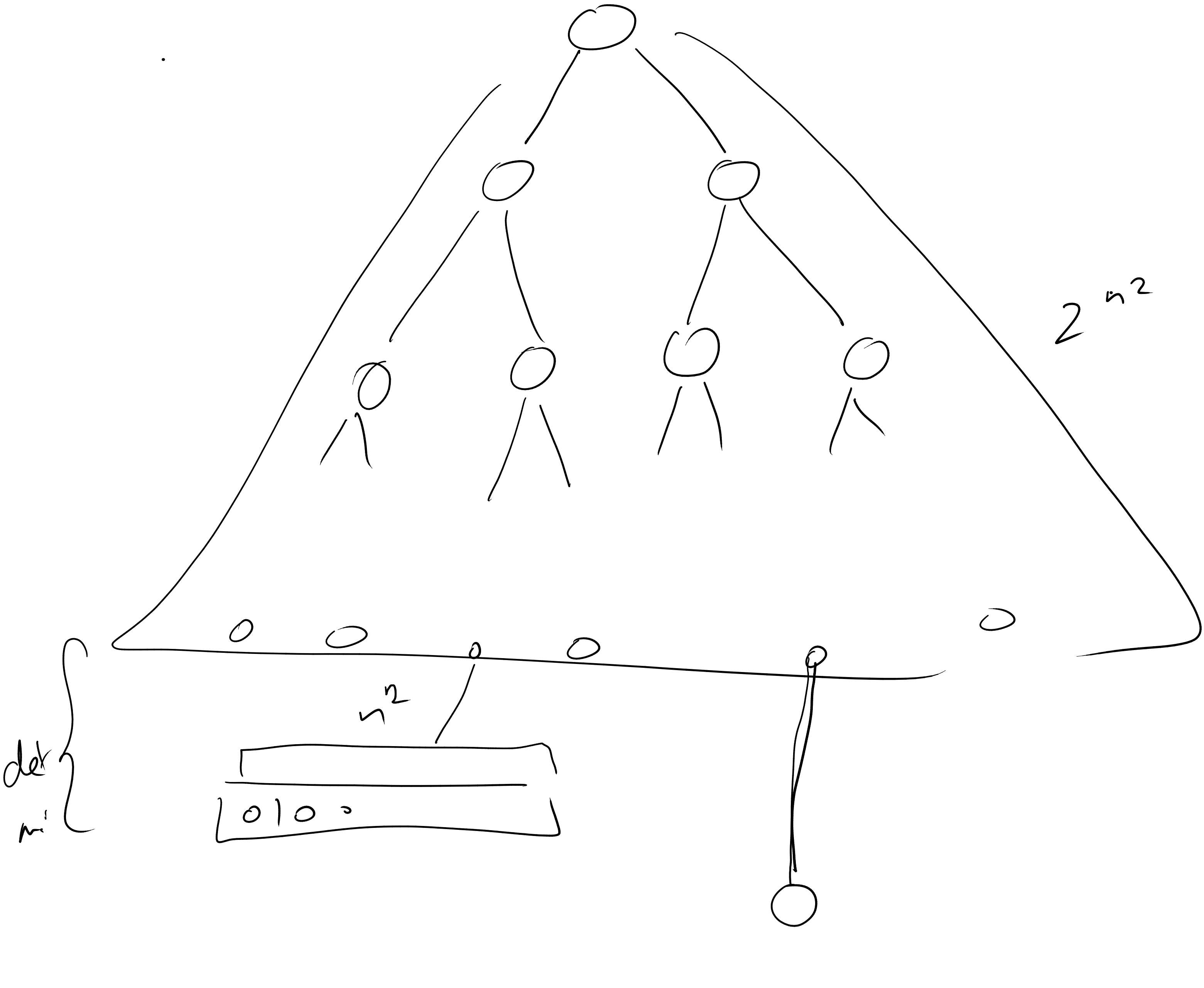
Set of all languages decidable by DTM in polynomial time

$$\boxed{NP = \bigcup_{k \in \mathbb{N}} NTIME(n^k)}$$

Set of all languages decidable by NTM in polynomial time.

$\text{EULER-PATH} \overset{\in P}{=} \{ G : \exists$ path that has every edge exactly once $\}$

$\underset{\substack{\cap \\ NP}}{\text{HAM}}\text{-PATH} = \{ G : \exists$ path that has every vertex exactly once $\}$



2^{n^2}
n^2
det
n!
0 1 0 0