Story So Far

$\Omega(n)$

- 1
  - Multiplication $O(n^2) \longrightarrow O(n \log n)$
  - Euler vs Ham Cycle / Path
  - Barber's Paradox / Hilbert's 10th Problem
- 2
  - Decision vs Search.
  - Encoding to Binary, String / Prefix free ✓
  - Turing Machine ✓
  - Encoding of Turing Machines ✓
- 3
  - Decidability
  - Halting Problem, Universal TM's ?
  - RE, Rec Languages
  - Robustness of TMs,
- 4
  - Nondeterminism
  - $\text{Reg} \subseteq \text{CFL} \subseteq \text{Rec}$

    DFA PDA TM's

- Complexity Classes DTIME, NTIME, DSPACE, NSPACE,
- P, NP

Euclid gcd $O(\log n)$

DFA = NFA = Reg. Languages

$\subsetneq$

PDA — Context Free Languages

$\subsetneq$

TM — Rec Langu.

$\subsetneq$

RE Langu.

Arithmetic hierarchy

$$\mathrm{DTIME}(n^k) \subseteq \mathrm{DSPACE}(n^k)$$

$$\cap \qquad\qquad \cap$$

$$\mathrm{NTIME}(n^k) \subseteq \mathrm{DTIME}(2^{O(n^k)})$$

Deterministic. $\delta : Q \times T^k \to Q \times T^k \times \{L, R\}^k$

Non Deterministic $\delta \subseteq Q \times T^k \times (Q \times T^k \times \{L, R\}^k)$

Function

Relation

$(q, a_1, \ldots a_k \to q', a_1' \ldots a_k', LLL)$

$(q, a_1 \ldots a_k, q', \ldots)$

$($ " $, q'' \ldots)$

NTM accepts x if $\exists$ exist a sequence of transition that leads to accepting state.

NTM rejects x if $\forall$ sequence of valid transitions should halt and lead to rejecting state.

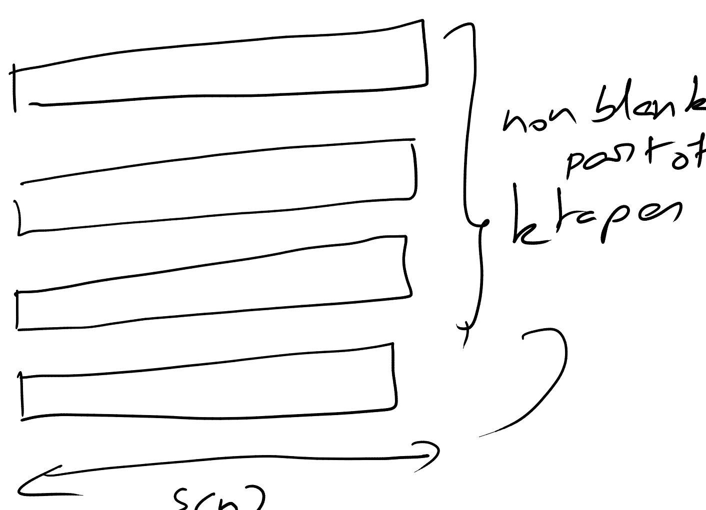
NTM decides L if

$\forall x \in L \Rightarrow M$ accept x

$\forall x \notin L \Rightarrow M$ rejects x.

Configuration of a TM

$(q, t_1, t_2, \ldots t_k)$



bits required to write configuration

$$\left(s(n) + \log|Q| + k \log s(n)\right) = \underline{O(s(n))}$$

Configuration Graph of a TM is Direct Graph.

$V =$ all the configuration of TM

E: $(C_1, C_2) \in E$ iff

in one step you can transition from

C_1 to C_2.

DTM: out degree of every vertex ≤ 1.

ie set of paths and cy.

C_{acc} | C_{rej}

NTM: out degree is bounded by a

$\leq |Q| \times |\Gamma|^k \times 2^k$

(independent of n)

C_{start} $(q_{st}, \triangleright x, \sqcup \sqcup \sqcup,)$

C_{acc} $(q_{acc}, \triangleright, \sqcup \sqcup \sqcup \sqcup)$

C_{rej} $(q_{acc}, \triangleright, \sqcup \sqcup \sqcup)$

$\boxed{\text{TM acc } x \iff C_{acc} \text{ is reachable from } C_{start}.}$

How big is the graph?

$\leq 2^{O(S(n))}$

Different Model of Computation

Circuits.

Is a directed acyclic graph, every node is labelled by $x_1, \dots x_n$, AND, OR, NOT

in deg (NOT) = 1 and a Root node.

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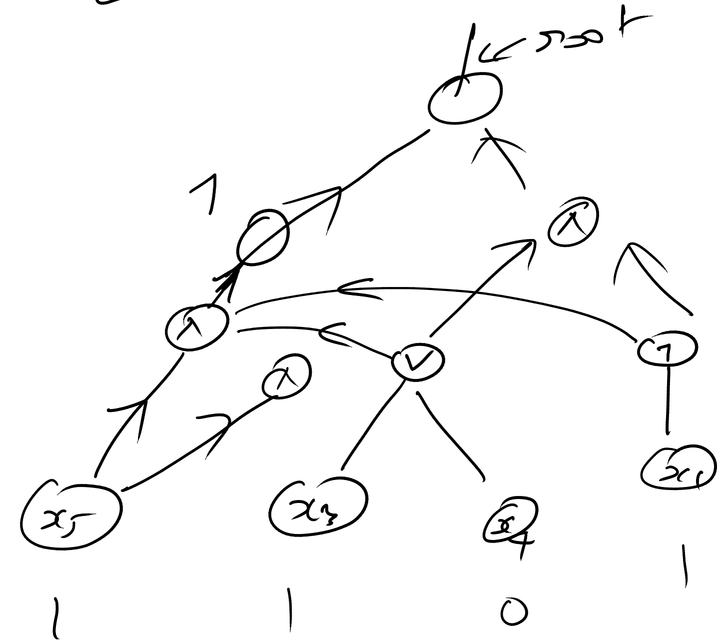


Evaluate Circuit on input.

\# nodes in circuit = size of circuit

- Circuit operates on a fixed length input
- For solving input of arbitrary size we will have a family of circuit.
  $\{C_i\}_{i \in \mathbb{N}}$ (can be a diff circuit for every size)
- Can Circuits decide all languages?
- $f : \{0,1\}^n \to \{0,1\}$ Boolean fn
- Shannon Circuit Lower bound.
  How functions are there?
  2^{2^n}
  How many circuit are there on n variables size $\leq t(n)$? ______

WLOG there is a unique node labeled x_i $\forall i$

Leaf nodes (index = 0)

are x_1 .. x_n

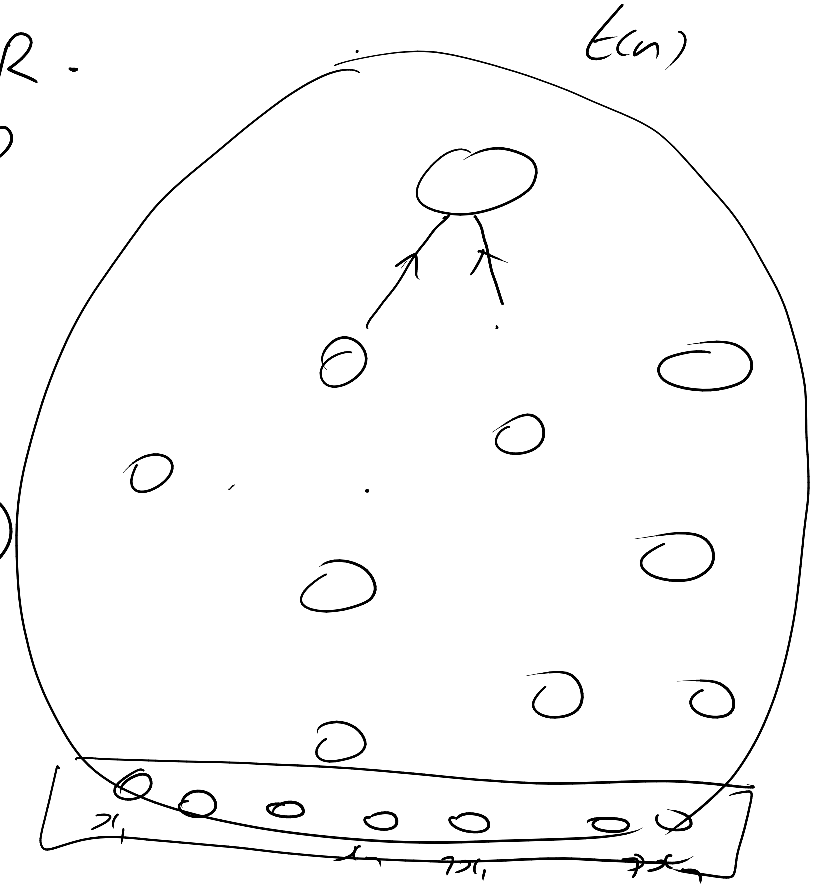
$\neg x_1$ $\neg x_n$

and every other node is labelled by AND or OR.

$$\le 2^{t(n)} \left(t(n) \cdot t(n) \right)^{t(n)}$$

first input 2nd input

$$\le 2^{t(n)} \, 2^{t(n) \cdot 2 \log(t(n))}$$



$$2^{2^n} \qquad 2^{O(t(n)\log t(n))}$$

If size is polynomial, cannot decide all fns.

$$2^n >> t(n)\log(t(n))$$

for $f(n) = n^k$

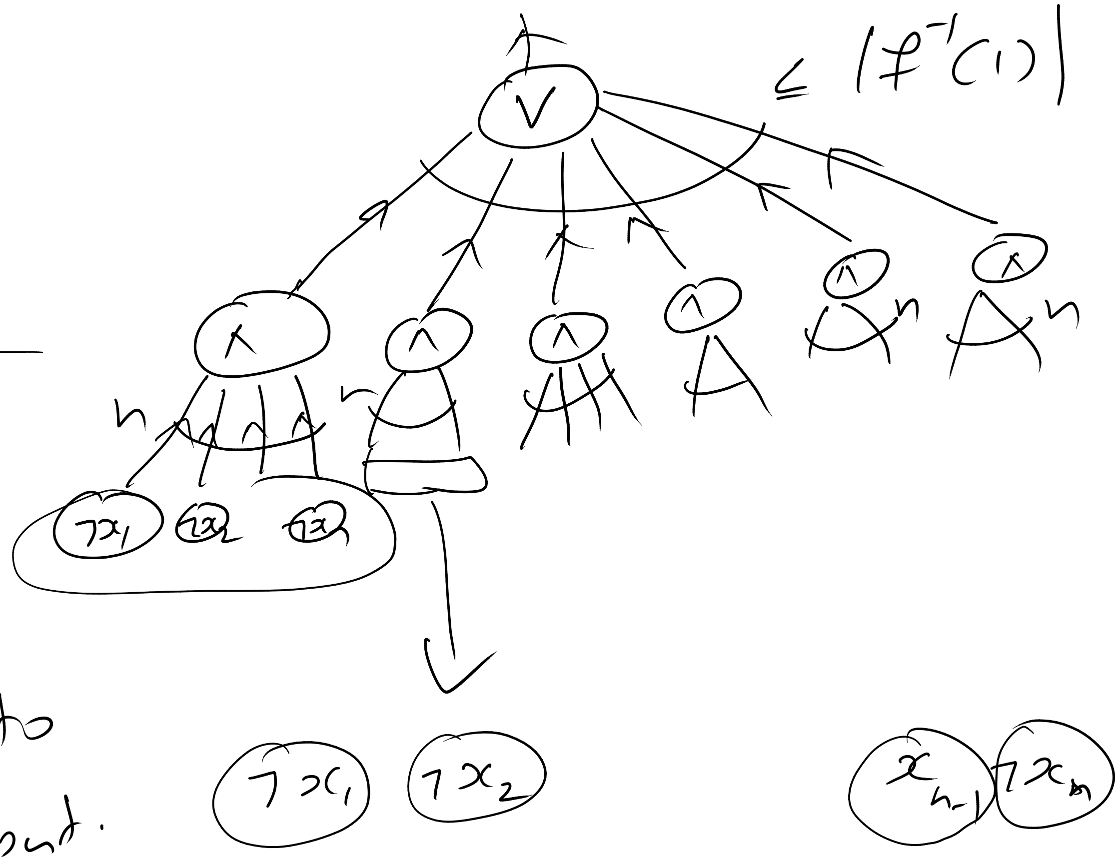
Is there a circuit for every function?

2^n

x_1						x_n	$f(x)$
0	0	0				0	1
							0
0	0	0	0	0	1	0	1
							1

Use truth table for building circuits?

$O(2^n)$



Correspond to every 1 input.

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$x_1 \wedge x_2 \wedge \neg x_3 \wedge x_4 \wedge \neg x_5 \wedge \neg x_6$

Formulae $\subseteq$ Circuit

Tree DAG