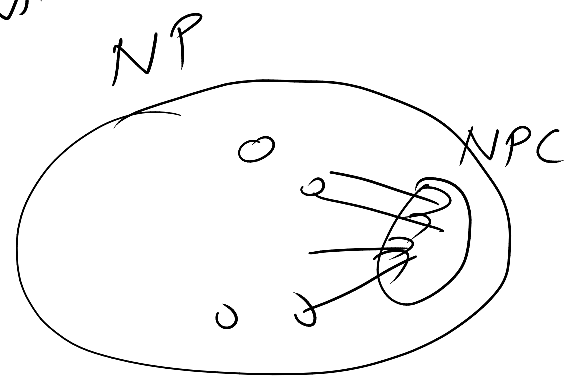
Previously.

- Reductions (Transitive)
- NP-Complete



- Cook-Levin Theorem

CNF (SAT)

$$(C_1) \wedge (C_2) \wedge \dots \wedge (C_m)$$

$$C_i = (x_1 \vee \neg x_4 \vee x_5 \vee \neg x_6)$$

CNF is NP-Complete.

T.S.T. $\forall L \in NP$,

$L \leq CNF$

M is the non-det poly time TM deciding L n^k

1. start configs

2. every consecutive configs are obtainable by δ-transitions

3. accepting configs

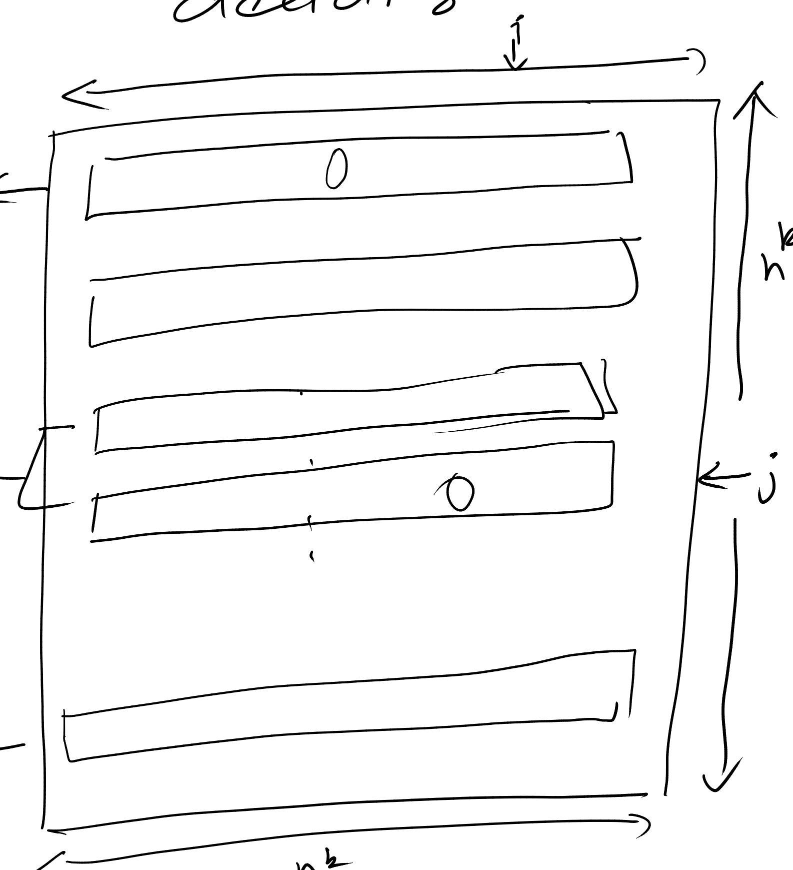


Tableau n^k

<u>Reduction (y)</u>

Construct a CNF formula φ

$$\varphi = \overset{①}{\varphi_{start}} \wedge \overset{②}{\varphi_{move}} \wedge \overset{③}{\varphi_{acc}} \wedge \overset{④}{\varphi_{cell}}$$

Tableau is a matrix of size $n^k \times n^k$
with entries in $Q \cup \Gamma$

x_{ijs} $\forall\, i, j \in \{1 .. n^k\}$, $\boxed{s \in Q \cup \Gamma}$

$x_{ijs} = 1$ iff cell $[i, j] = s$

variables $= O(\underline{n^{2k}})$

(4) $\forall\, i, j$
Exactly one of x_{ijs} is $\underline{1}$
(for $s \in Q \cup \Gamma$)

$\varphi_{cell} =$ $f : \{0,1\}^{|Q \cup \Gamma|} \longrightarrow \underline{\{0,1\}}$

$$\varphi_{start} = x_{11\triangleright} \wedge x_{11q_{st}} \wedge x_{12y_1} \ldots \wedge x_{1ny_{n-1}} \wedge x_{1\,n+1\,y_n}$$

$$\varphi_{acc} = x_{11q_{acc}} \vee x_{12q_{acc}} \ldots \vee x_{1n^k q_{acc}}$$

φ_{move}: every row is a config that yields a config on the next row by δ transition

$$\varphi_{move} = \underbrace{\bigwedge_{i,j}}_{n^{2k}} \Big(\bigvee_{\substack{a_1 \ldots a_6 \in S \\ \text{valid}}} (x_{ija_1} \wedge x_{ij+1a_2} \ldots) \Big)$$

2×3 assignments to windows.

a_1	a_2	a_3
a_4	a_5	a_6

$S \subseteq (Q \cup \Gamma)^6$.

<u>Claim:</u>

<u>$x \in L$</u> $\iff$ φ_x is satisfiable.

More Reductions:

Vertex Cover: $G(V, E)$

a vertex cover is a set $S \subseteq V$ s.t $\forall$ every edge $e \in E$, has one endpoint in S.

$$\text{VERTEX-COVER} = \{\langle G, k\rangle : G \text{ has a V.C. of size } \le k\}$$

Claim: $\underbrace{\text{VERTEX}}_{}$-COVER $\in$ NP-Complete

VC

Proof:

① $VC \in NP$

$T \subseteq V$

$V(\langle G, k\rangle, \langle T\rangle)\{$

- Check if $\forall e \in E$, $e = (u, v)$, $u \in T$ or $v \in T$
- Check $|T| \le k$

$\}$

② $L \leq VC$.

$L \Rightarrow$ CNF $\longrightarrow$ INDSET $\longrightarrow$ VERTEX COVER

INDSET

$\langle G, k \rangle$

there is $S \subseteq V$

$|S| > k$, s.t

all edges are outside S

VERTEX COVER

$\langle H, k' \rangle$

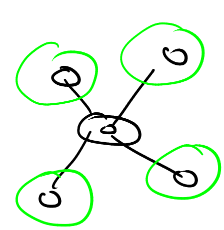
there is $S \subseteq V(H)$

$|S| < k'$, s.t.

S touches all edges

$$\boxed{\begin{aligned} H &= G \\ k' &= n - k \end{aligned}}$$

- Complement of an I.S. is a Vertex Cover.



INTEGER-PROGRAMMING

$$\left[\begin{array}{l} x_1 + 4x_2 + 5x_3 \geq 25 \\ x_2 - 3x_4 + x_1 \leq 5 \\ \\ \\ \end{array}\right] m$$

Find integer $x_1 \ldots x_n$ that satisfies the m inequalities.

LINEAR-PROGRAMMING $\in P$

$x_1 .. x_n \in \mathbb{R}$

(Karmarkar)

INT. PROG $\in$ NP-COMPLETE

$3CNF \leq INT.PROG.$

$\phi = C_1 \wedge \cdots \wedge C_m$

$C_i = (x_1^i \vee x_2^i \vee x_3^i)$

n-variables and m inequalities.

$x_i \longrightarrow v_i$

$\neg x_i \longrightarrow (1 - v_i)$

$(x_1 \vee x_2 \vee \neg x_3)$

$\Rightarrow v_1 + v_2 + (1 - v_3) \geq 1$

$\boxed{v_i \leq 1, \quad v_i \geq 0}$

ϕ

# of inequalities $= m + 2n$

(1.) Reduction is polytime. (check)

(2.) $\phi \in CNF \iff \phi \in INT\text{-}PROG.$

HAM-PATH

$\langle G \rangle$ has a HAM-tour.

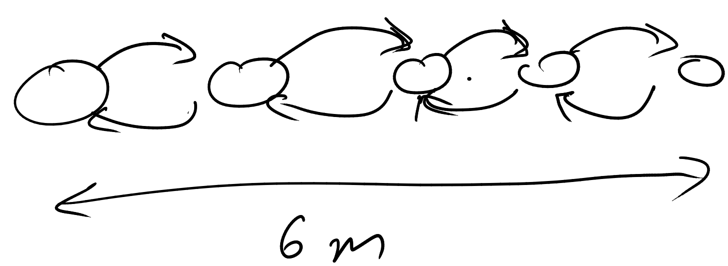
3CNF $\longrightarrow$ HAM PATH

$\varphi = C_1 \wedge \dots \wedge C_m$ $\qquad$ G

Goal: To construct G s.t.,

φ is satisfiable $\iff$ G has a Ham-tour.

(1.) Each x_i



(2.) v_{start}
- no incoming.