2.1

a) $M^T v = \lambda v$

$$\begin{bmatrix} p_1 & \cdots & p_n \\ & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} v_1 \\ \\ \\ \vdots \end{bmatrix} = \lambda \begin{bmatrix} v_1 \\ \\ \\ \\ \end{bmatrix}$$

$$|\lambda||v_1| \le \sum_{i=1}^{n} p_i |v_i| \le |v_1| \qquad p_i \ge 0$$

$$\sum p_i = 1$$

$$|\lambda v_1| = \left|\sum_{i=1}^{n} p_i v_i\right|$$

$$\le \sum_{i=1}^{n} p_i |v_i|$$

$$|\lambda| \, |v_1| \le |v_1|$$

$$|\lambda| \le 1$$

b.)

$$v_a = \frac{d_a}{\sum_{b \in V} d_b}$$

$$(Mv)_a = \sum_{b=1}^{n} M_{a,b} v_b = \sum_{b \in N(a)} \frac{1}{\cancel{d_b}} \frac{\cancel{d_b}}{\sum_{b' \in V} d_{b'}} = \frac{d_a}{\sum_{b' \in V} d_{b'}}$$

---

2 c)

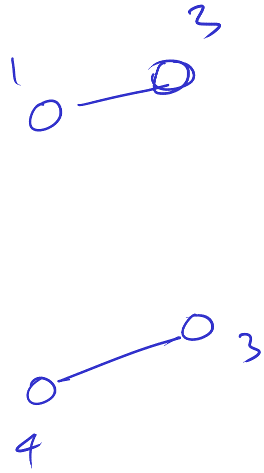
r = # of connected components.

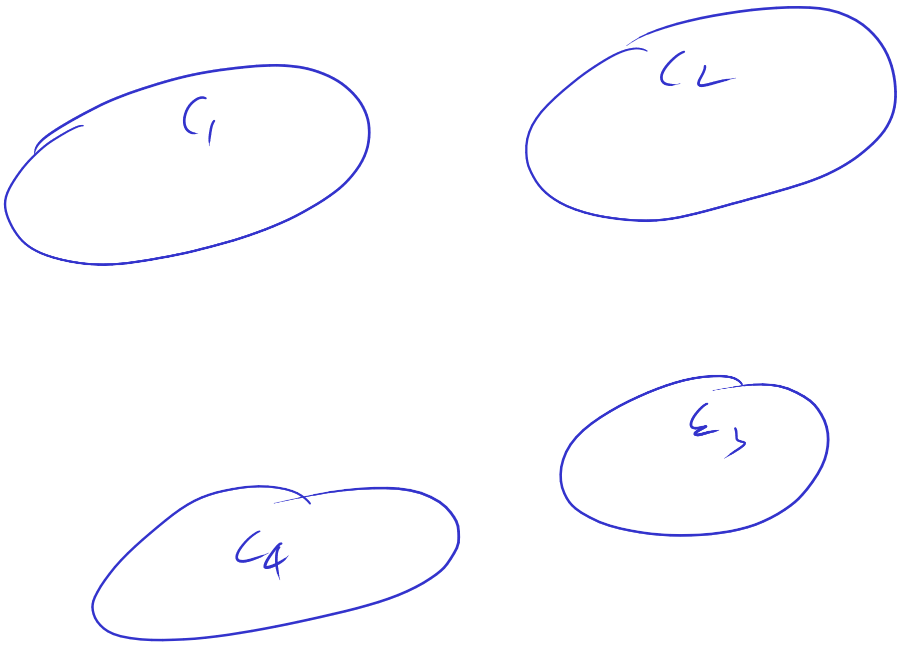
r' = # l.i. e.v with e.v. 1

$\underline{r \leq r'}$

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \begin{bmatrix} \Pi_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ \Pi_2 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Pi_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & \\ 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$





$g' \le g$

$$\mathrm{geom}_M(1) = \sum_{i=1}^{k} \mathrm{geom}_{M_i}(1)$$

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$v_1 \quad u_1$

$v_2 \quad u_2 \quad w_2$

$v_3 \quad u_3$

$$\begin{bmatrix} v_1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow v_1$ M e.v of M_i

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

We need to show that

If G is connected then

$\text{geom}(1) = 1$

Proof:

v, u are l. ind.

and $Mu = u$, $Mv = v$

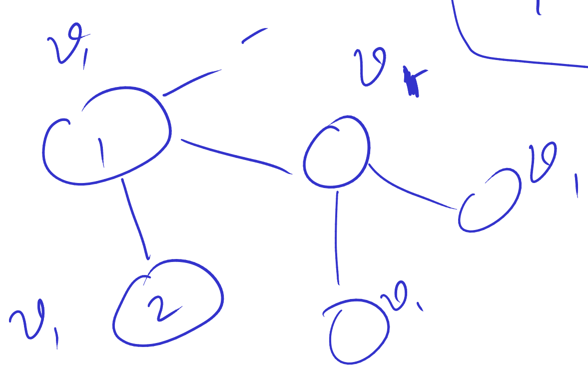
Claim

$$\text{geom}_M(1) = \text{geom}_{M^T}(1) = 1$$

$$\begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_1 \\ \vdots \\ v_1 \\ v_1 \\ v_1 \end{bmatrix} = v_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

M^T

$$\boxed{v_1 = \sum_{i=1}^{n} p_i v_i}$$

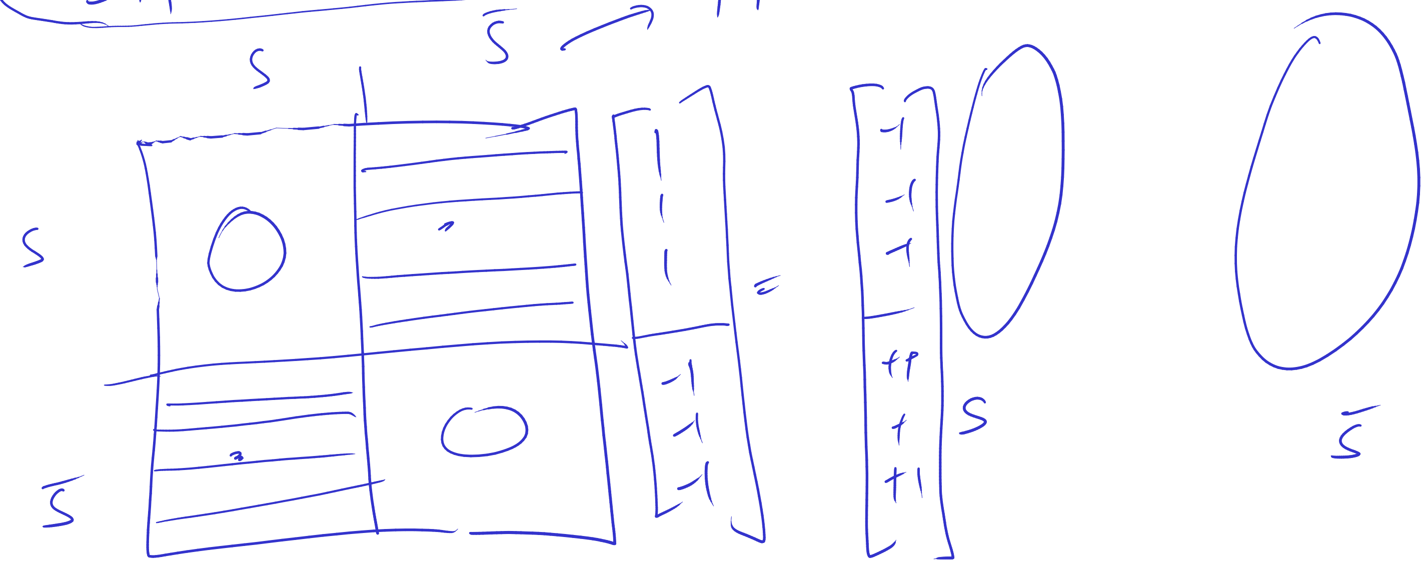


$$\mathrm{geom}_M(1) = \sum_{i=1}^{k} \mathrm{geom}_{M_i}(1)$$
$$= \#\ \text{connected components.}$$

2.1

d.)

bipartite $\Rightarrow$ e.v. $= -1$



e.v $= -1$ $\Rightarrow$ bipartite ✓

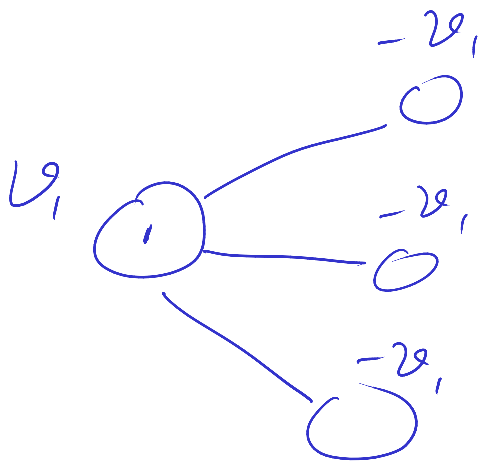
$$\boxed{\begin{aligned} &\mathrm{trace}(ABC) \\ &= \mathrm{trace}(CAB) \\ &= \mathrm{trace}(BCA) \end{aligned}}$$

$$\begin{aligned} \mathrm{trace}(M^{-1}) &= \mathrm{trace}(P^{-1}D\,P) \\ &= \mathrm{trace}(P\,P^{-1}D) \\ &= \mathrm{trace}(D) \end{aligned}$$

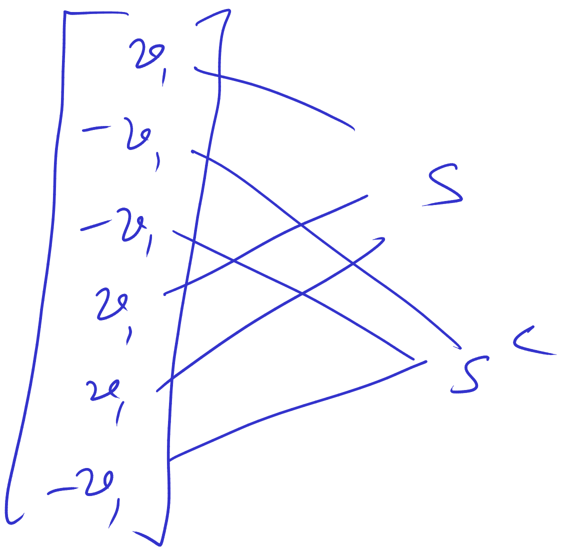
$$-v_1 = \sum_{i=1}^{n} p_i v_i$$

v_1 is largest in abs value.

all nonzero v_i's $= -v_1$



$$\begin{bmatrix} v_1 \\ v_1 \\ v_1 \\ -v_1 \\ v_1 \\ -v_1 \\ -v_1 \end{bmatrix}$$



2.3 a)

$$\det(A - \lambda I)$$

$$= \det((A - \lambda I)^T)$$

$$= \det(A^T - \lambda I)$$

b)

$$D = P^{-1} M P$$

$$\boxed{\begin{array}{l} v \longrightarrow M \\ P^{-1} v \longrightarrow M' \end{array}}$$

$$\det(AB) = \det(A)\det(B)$$

$$\det(D) = \det(M)$$