Linear Algebra

Set of all functions

$$\{ f : \{0,1\}^n \longrightarrow \mathbb{R} \} = F$$

field

A set S in a vect space over a field $\mathbb{F}$ iff

- $\forall \alpha, \beta \in \mathbb{F}, \forall f, g \in S$

$$\alpha f + \beta g \in S$$

$1 \cdot f = f$

$0 \cdot f = 0$

$(f+g)(x) = f(x) + g(x)$

$\alpha \cdot f + (-\alpha) \cdot f = 0$

$1 \cdot f_{(x)} \oplus 1 \cdot f_{(x)} =$

F is a vectorspace over $\mathbb{R}$

$\dim(F) = 2^n$

$f: \{0,1\} \to \mathbb{R}$

$f(0) = a \qquad f(1) = b$

$f(0) = 1 \qquad f(1) = 0$

$g(0) = 0 \qquad g(1) = 1$

$h(0) = a \quad , \quad h(1) = b$

$h = af + bg$

$\mathcal{F} \longrightarrow \mathbb{R} \times \mathbb{R}$

$\mathcal{F}$ is a vector space over $\mathbb{R}$

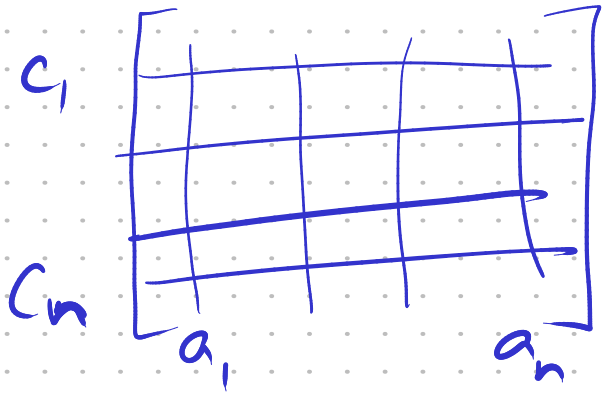
Linear Transform and Matrix

$T: V \to V$ (operators)

With respect to basis $\{b_1 .. b_n\}$

M is matrix of T

M^t

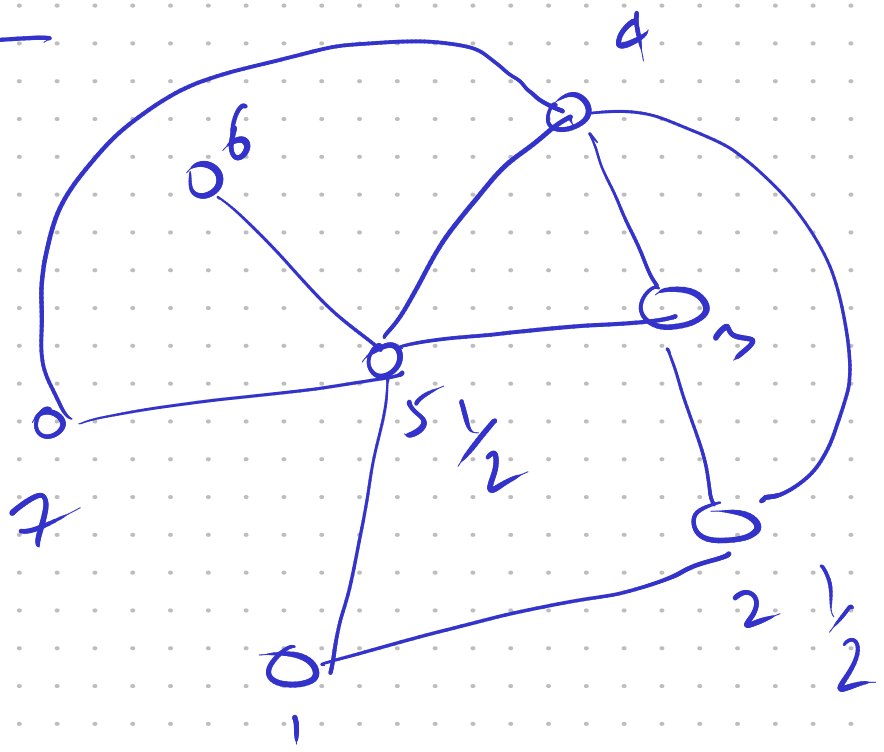


$\dim(\text{Rang}(M))$

$= \dim(\text{span}(a_1 .. a_n))$

$\dim(\text{span}(c_1 .. c_n))$

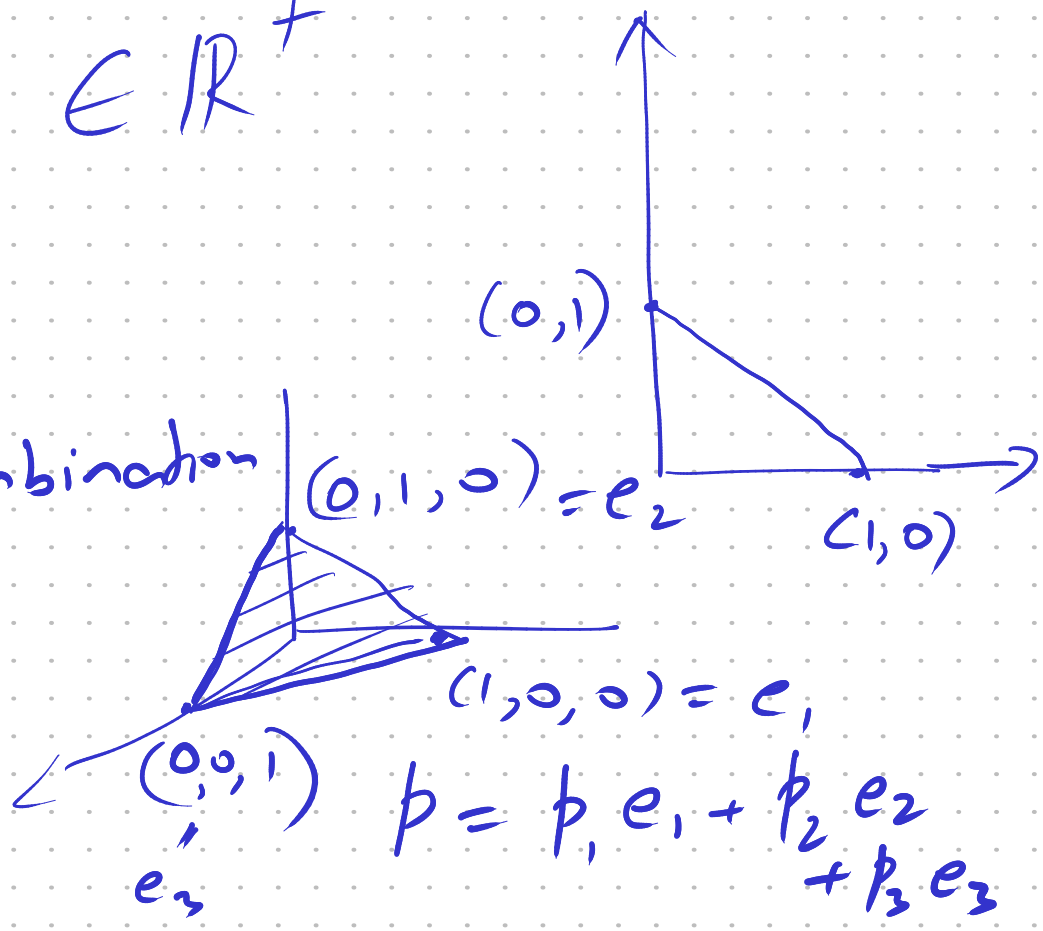
Operadores



$$p^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^7$$

Convex combination

- $p_i \geq 0$
- $\sum p_i = 1$



Set prob dist $\subseteq \mathbb{R}^7$

Convex Combinations of $\subseteq \mathbb{R}^7$

$e_1 \cdots e_7$

p^0 $\quad p^1 = \begin{bmatrix} 0 \\ 1/2 \\ 0 \\ 0 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$ $\quad p^2 = \begin{bmatrix} 1/2 \cdot 1/3 + 1/2 \cdot 1/3 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$

M

$$\begin{bmatrix} 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 \\ 0 & 1/3 & 1/3 \\ 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

$$p^{(t+1)} = M p^t \Rightarrow p^{t+1} = M^{t+1} p^0$$

p^t is distribution after t random steps from init. dist p^0

What happens after a long time?

ie $t \to \infty$?

$$p^{t+1} = p^{t+2} \quad \text{(at equilibrium)}$$
$$= M p^{t+1}$$