Singular Value Decomposition

Suppose $A \in \mathbb{R}^{n \times m}$ $\quad n \leq m$

Let $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_r > 0$ be eigenvalues of $AA^T \in \mathbb{R}^{n \times n}$ $\quad r = \text{rank}(A)$

$v_1 \quad v_2 \ldots v_r \quad v_n$ O.N.E.V.B

$$w_1 = \frac{Av_1}{\sqrt{\lambda_1}} \quad \ldots \quad w_n = \frac{Av_n}{\sqrt{\lambda_n}} \ldots w_m$$

→ will for O.N.E.V for A^TA

and

$$A = \sum_{i=1}^{r} \sqrt{\lambda_i}\, v_i w_i^T$$

r → singular values

v_i → singular vectors

$n \times m$ entries $\geq$ $r(1+n+m)$

$r = 3$

$\lambda_1 \geq \lambda_2 \geq \lambda_3$

$$A = \sqrt{\lambda_1}\, v_1 w_1^T + \sqrt{\lambda_2}\, v_2 w_2^T + \sqrt{\lambda_3}\, v_3 w_3^T$$

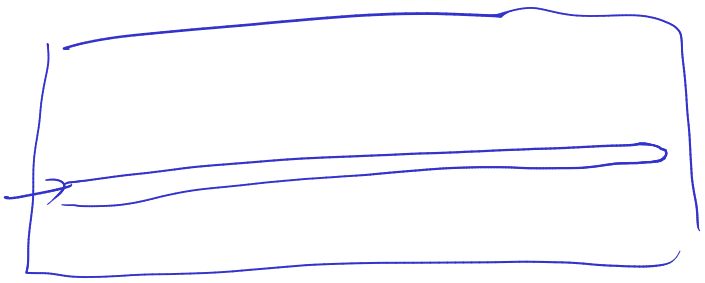
red → $v_1 w_1^T$; green → $v_2 w_2^T$; black → $v_3 w_3^T$

v_1 = dir of the best fit line

plane of v_1, v_2 = best fit plane for the galaxy.

$n = 32$

$m = 83781$

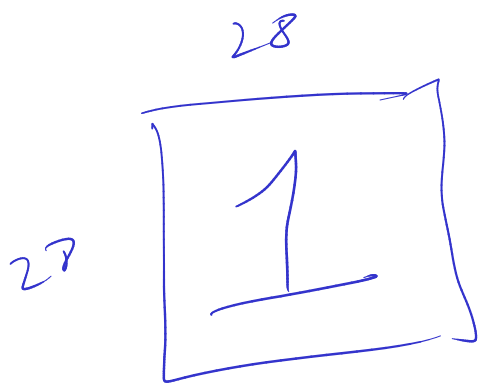


$$\sum_{i=1}^{32} \sqrt{\lambda_i}\, v_i w_i^T \approx \sum_{i=1}^{4} \sqrt{\lambda_i}\, v_i w_i^T \quad (4 \to 10\text{–}15)$$

$32 \times 83781 \Rightarrow 4(1 + 32 + 83781)$

MNIST

28×28 b/w of handwritten digits.



50,000 / 28*28

Problem

Can we visualize these data points in a sensible way?

28*28 = 768