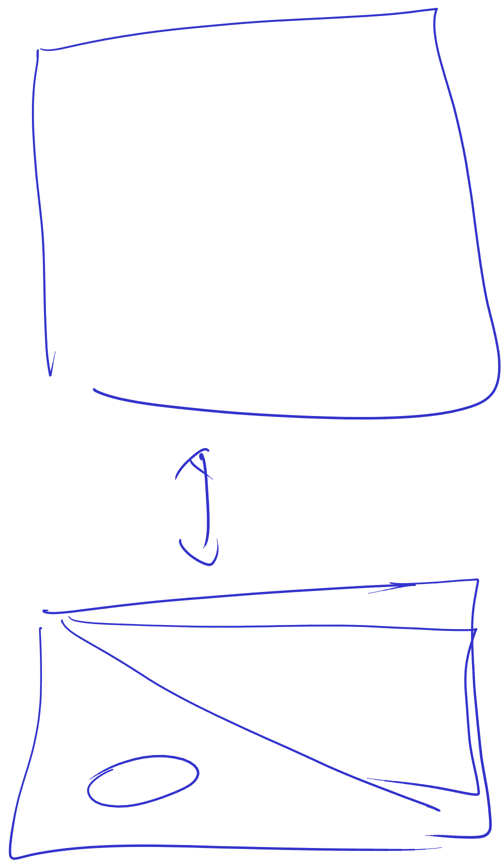
1. Linear Equation

$$ax + by = c$$



$$Mx = b$$

2. Vector Space (over a field $\mathbb{F}$)

$+ : V \times V \to V$

$* : V \times \mathbb{F} \to V$

- Basis Vectors.

Every vector can be rep uniquely interm of the basis vectors.

$$|B| = |B'| = \dim(V)$$

$n \qquad n+1$

3. Linear Transformation

$T : V \rightarrow W$

Given a basis for V, W, there is a matrix M

$$T(\alpha v + \beta w) = \alpha T(v) + \beta T(w)$$

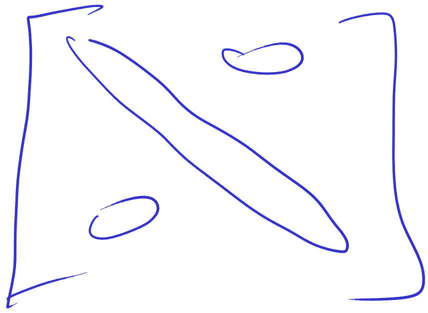
- Kernel and Range.

$$\dim(K) + \dim(R) = n$$

- $\mathrm{rank}(M) = \dim(\mathrm{Range})$

- Change of basis

V, W



$$M' = \underbrace{B^{-1} M B}$$

$T: \underbrace{V \longrightarrow V}$ (operator)

$$M' = P^{-1} M P$$

"Choosing the right basis to study some set L.T.'s"

④ Determinants.

$$\det\left(\begin{bmatrix} u \\ v \end{bmatrix}\right) =$$

Parallelopiped

5. E.V.

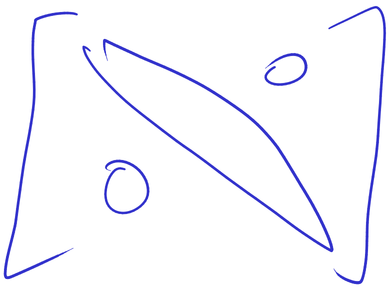
$$Mv^1 = v^2$$

$n \times n$

$$\boxed{Mv = v}$$

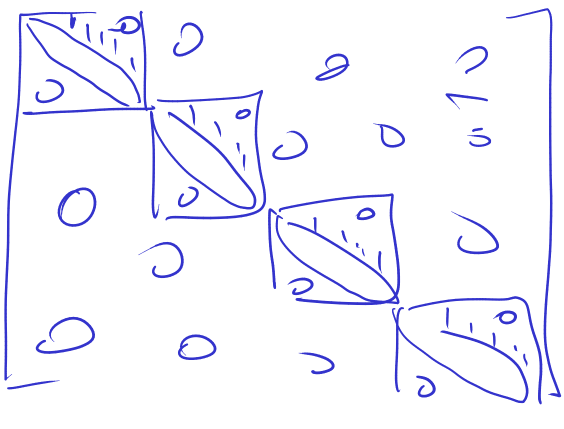
$$\underbrace{M}_{} \underbrace{v}_{\text{eigenvector}} = \underbrace{\lambda}_{\text{eigenvalue}} v$$

e.vect of diff e.vals are lin. indep.



in the e.vect basis.

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$



Char Poly = det$(M - \lambda I)$ var.

alg. multiplicity = # repeated root for λ

geom " = $\dim(\ker(M - \lambda I))$

$= \dim(\text{EigSpace}(\lambda))$

$Mv = \lambda v$

6. Norm and Inner Product

Orthonormal vectors.

7. Symmetric Matrices

- eivals are $\mathbb{R}$
- e.vect for diff λ are orthogonal

M symmetric $\iff$ M has O.N.B.EV

$P^{-1}MP = P^{T}MP$

Symmetric $\longrightarrow$ Hermitian

$M^T = M$ $\qquad\qquad$ $\bar{M}^T = M$

Orthogonal $\longrightarrow$ Unitary

$P^T = P^{-1}$ $\qquad\qquad$ $\bar{P}^T = P^{-1}$

$$M = \sum_{i=1}^{n} \lambda_i \, v_i v_i^T$$

8. SVD

$$M = \sum_{i=1}^{n} \sqrt{\lambda_i} \, v_i w_i^T$$

SVD and Best Fit Subspace.

Error Correcting Codes

subspace $\subseteq \mathbb{F}_p^n$

$\mathbb{F}_p = \{0, \ldots \quad p-1\}$ $\overset{+}{\times}$ in mod p