

- Eigen values and eigen vectors
- Eigen. vectors corresponding to distinct eigen values are linearly independent. ✓
- # eigen values $\leq n$

$$\det(M - \lambda I) = 0$$

- $M \in \mathbb{R}^{n \times n}$
 → poly. over real field # $\leq n$
 → poly. over \mathbb{C} field # roots = n

Suppose M had n linearly independent eigen vectors

$$\{v_1, \dots, v_n\} \quad \lambda_1, \dots, \lambda_n \quad e_i = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$w = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

$$Mw = \lambda_1 \alpha_1 v_1 + \lambda_2 \alpha_2 v_2 + \dots + \lambda_n \alpha_n v_n$$

What is M in $\{v_1, \dots, v_n\}$ basis?

$$\begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix} = P^{-1} M P$$

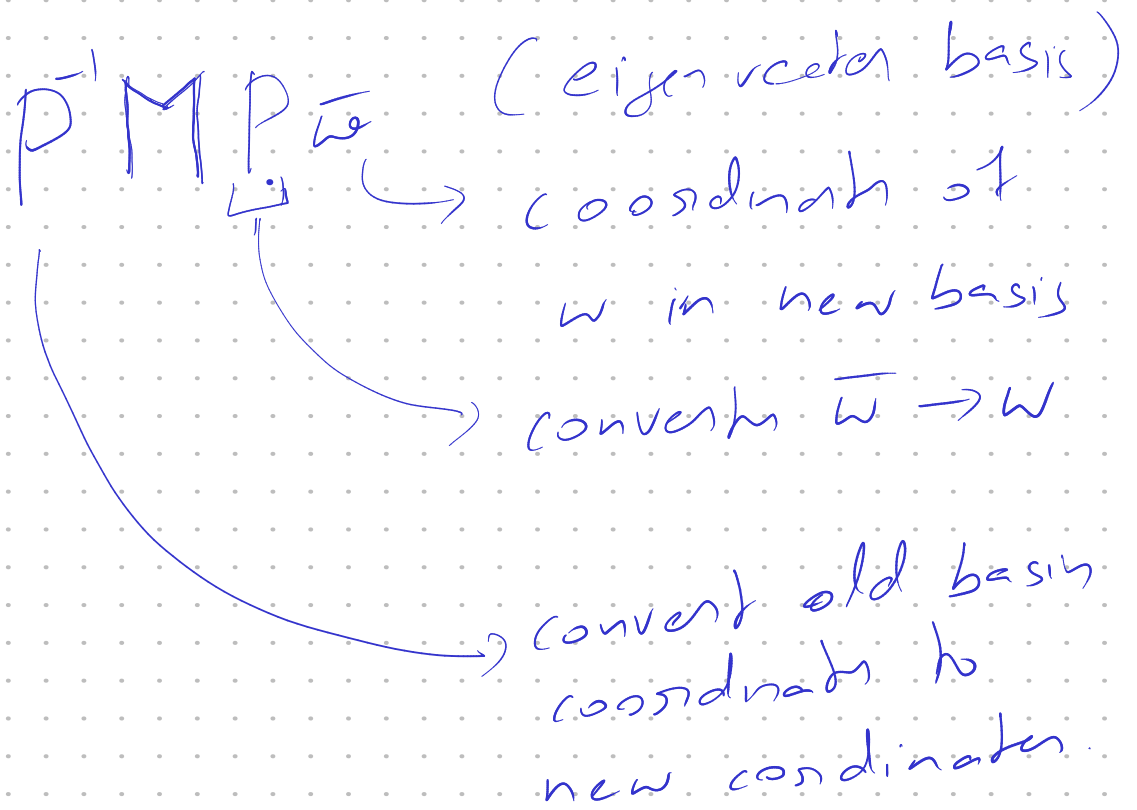
where P is an invertible matrix

$$T: V \rightarrow V$$

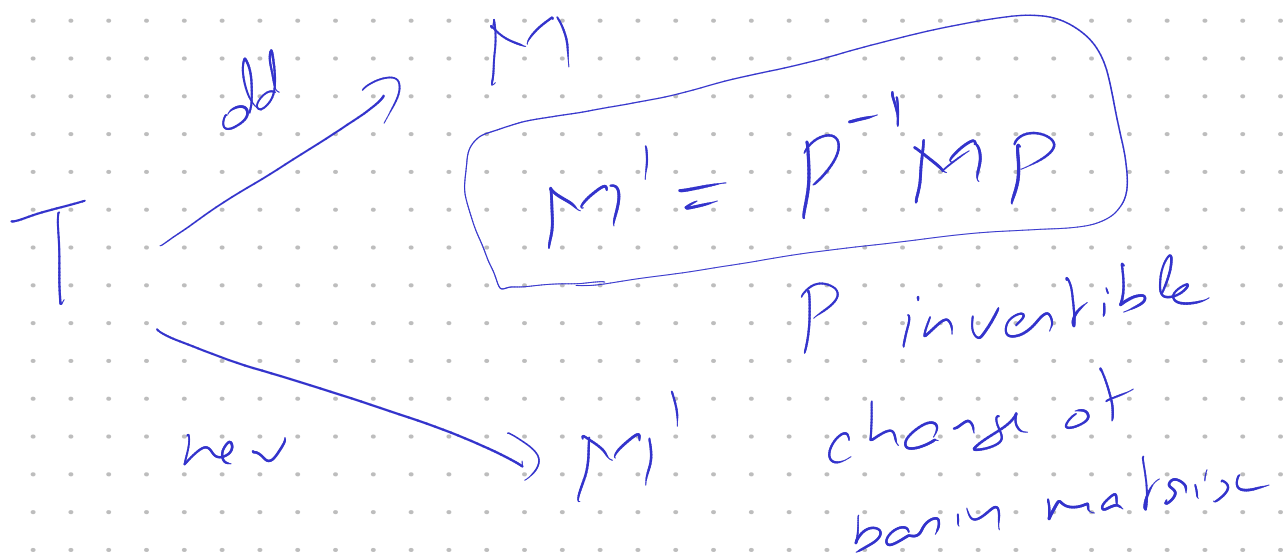
$$T: V \rightarrow W$$

$$B = \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \dots & v_n \\ | & | & & | \end{bmatrix}$$

Mw (original std basis)



Hence $P^{-1}MP$ is matrix in new basis



If P corresponds to change of basis
to eigen vector basis, then
 M' is a diagonal matrix

What happens when there are no
 n linearly indep eigen vectors?

Equivalence relation

$$M \equiv M' \text{ iff } \exists \text{ invertible } P \text{ such that } M' = P^{-1} M P$$

If $M \equiv$ to a diagonal matrix,
then M is diagonalizable

Are there matrices that are not
diagonalizable?

$$M'' = Q^{-1} M' Q$$

$$M'' = Q^{-1} P^{-1} M P Q$$

eigen value can be 0

If M is non invertible then

M has eigen value of 0

v_1, v_2, v_3
↑
null space

v_n

$$\begin{bmatrix} 0 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$$\det(M - \lambda I) = 0$$

over \mathbb{C} field

characteristic polynomial of M

$$\prod_{i=1}^n (\lambda - \alpha_i) = 0$$

$$\alpha_1 = \alpha_2 = \alpha_3 = a$$

$$\alpha_4 = \alpha_5 = b$$

$$\alpha_6 = \alpha_7 = \alpha_8 = c$$

of term with root α is called

algebraic multiplicity

geometric multiplicity of a = maximal # of linearly independent eigenvectors with eigenvalue a

$$n=8$$

$$(x-a)^3 (x-b)^2 (x-c)^3 = 0$$

linearly independent eigenvectors with eigenvalue $a = 3$

$$\gg b = 2$$

$$\gg c = 3$$

If algebraic multiplicity = geometric multiplicity \rightarrow then matrix is diagonalizable

But there are matrices s.t.

alg \neq geo multiplicity.

So all matrices are not diagonalizable

$$\text{Eigen-Space}_M(\lambda) = \{v \in V: Mv = \lambda v\}$$

Is it a subspace?

v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8

