Are all matrices diagonalizable?

No

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \longrightarrow P^{-1} M P$$

eigen values 0

eigen vector $\begin{bmatrix} \alpha \\ 0 \end{bmatrix}$ $\alpha \neq 0$

no lin indep eigenvectors.

Char. poly $\det(M - \lambda I) = 0$

$$(\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n) = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 \neq \lambda_4 = \lambda_5 \neq \lambda_6 \cdots$$

alg. multiplicity $(\lambda_1) = 3$

geometric multiplicity(λ)

$= \dim(\ker(M - \lambda I))$

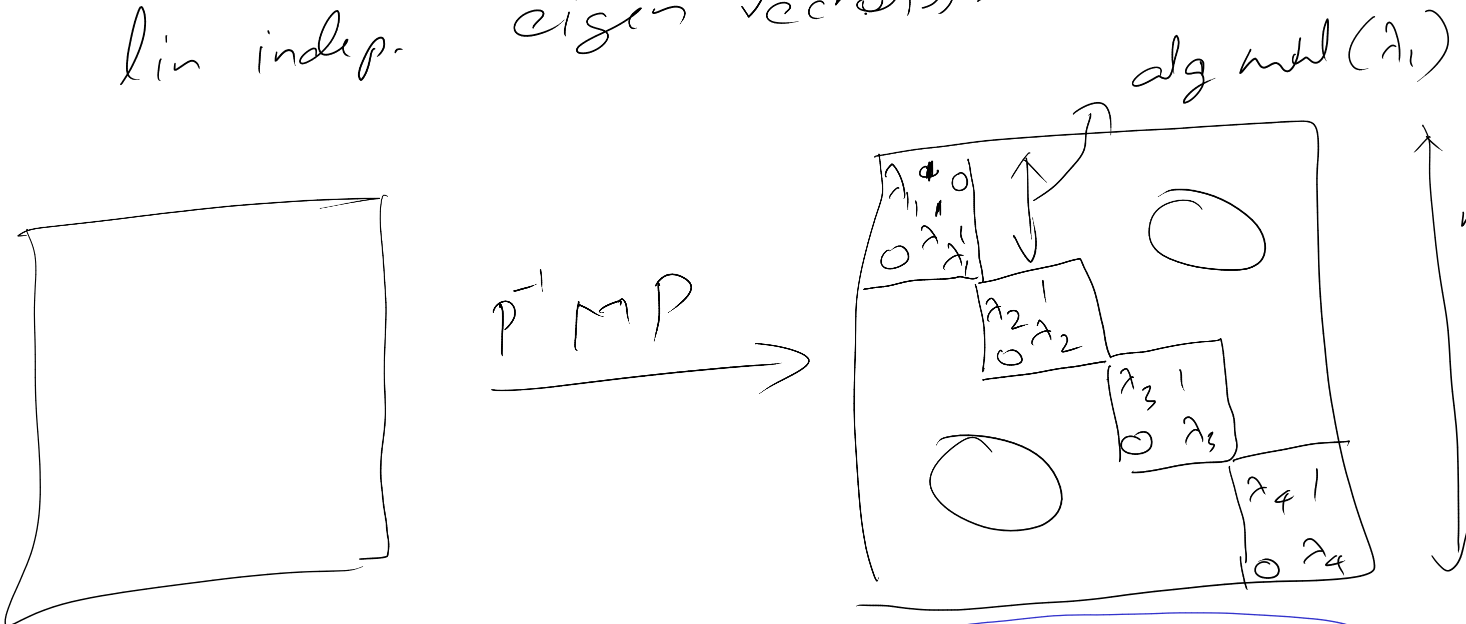
$= \#$ lin. indep. eigenvect with eigen value λ

alg $\neq$ geom multiplicity.

$\boxed{\text{geom} \leq \text{alg.}}$

Generalized eigen vectors

For every M there are n generalized lin indep. eigen vectors.



Jordan Form

Down with Determinants

Norms and Inner Products

Norm of v "length of v" L_2

$$\|v\| = \sqrt{v_1^2 + v_2^2 \cdots + v_n^2} \qquad v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$\mathbb{R}^n$ – Euclidean Vector Space

$$\|v\|_p = \left(|v_1|^p + |v_2|^p \cdots + |v_n|^p \right)^{1/p}$$

p^{th} – norm $\quad L_p$

$$\|v\| = \|v\|_2 \qquad \|v\|_1$$

$$\|v\|_1 = \left(|v_1| + |v_2| \cdots + |v_n| \right)$$

$\|v\|_0 :=$ # non zero entries in v

$M \qquad M_{ij} = i^{th}$ row, j^{th} column entry.

In general, $\| \ \| : V \to \mathbb{R}$ satisfies

- $\|v\| \geq 0 \ \forall v \in V$
- $\|v\| = 0$ iff $v = 0$
- $\forall v, w \in V$,

$$\|v + w\| \leq \|v\| + \|w\|$$

$$\text{\rotatebox{90}{$\leq$}}$$

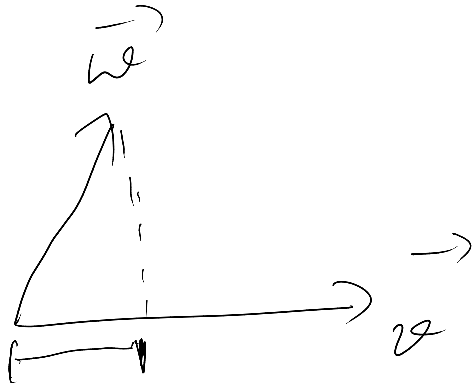
$$\|v\| - \|w\|$$

Distance between vectors

$$\text{dist}(v, w) = \|v - w\|$$

Angle, Projections

$$\frac{\vec{w}\cdot\vec{v}}{\|\vec{v}\|}$$



dot product ($\mathbb{R}$ - vector space)

$$\vec{w}\cdot\vec{v} = \sum_{i=1}^{n} w_i \cdot v_i$$

$$\vec{w}\cdot\vec{v} := \sum_{i=1}^{n} w_i \cdot \overline{v_i} \quad (\mathbb{C}\text{- vector space})$$

In general, an inner product is function $\langle\ ,\ \rangle : V\times V \longrightarrow \mathbb{R}, \mathbb{C}$

- $\langle v, v\rangle \geq 0 \quad \forall v \in V$
- $\langle v, v\rangle = 0 \quad \text{iff } v = 0$
- $\langle v+v', w\rangle = \langle v, w\rangle + \langle v', w\rangle$
- $\langle av, w\rangle = a\,\langle v, w\rangle$
-

- $\langle v, w \rangle = \overline{\langle w, v \rangle} \quad \forall\, v, w \in V$

  (conjugate symmetry)

Norm based on inner product

$$\|v\| := \sqrt{\langle v, v \rangle}$$