Probability and Statistics

Quiz 2

Directions

- Leave the bag/phone in front or at the back of the hall. Only pen can be taken to the seat.

- Find a nearby seat where answer sheets are kept. Write name, roll no, at the top of the sheet.

- Stop writing at 940AM.

Q1. A rat runs through the maze shown on the right. At each step it leaves the room it is in by choosing at random one of the doors out of the room.

(a) Give the transition matrix P for this Markov chain.

(b) Show that it is irreducible (has a single recurrent class) but not aperiodic.

(c) Find the stationary distribution.

(d) Find the expected time for it to return to Room 1 after starting from there.

(e) Now suppose that a piece of cheese is placed on a deadly trap in Room 5, find the probability that the mouse gets caught in the trap at an even time and the probability that it get caught in odd time (ie T the time at which mouse gets caught is even or odd).
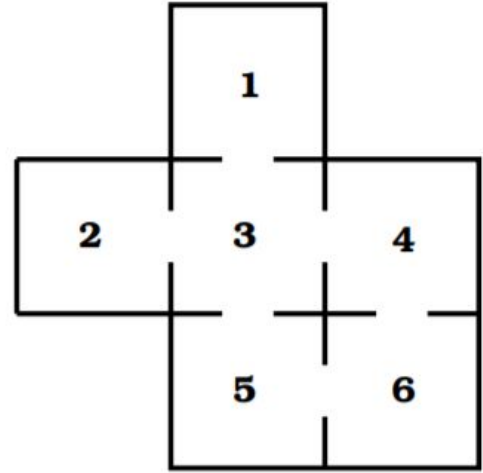
(For the last 2 problems, your answer could be a system of linear equations and just describe how to obtain the answer from the solution to the system)

(1 mark for each)

Q2. Consider a Poisson process with rate $\lambda=0.5$. (can leave the solution in terms of exponentials)

(a) What is the probability that the third arrival occurs after 8 time units? *(1.5 marks)*

(b) What is the probability of at most 1 arrival in $[0, 8)$ and at least 2 arrival in $[8, 16)$, and at most 1 arrival in $[16, 24)$. *(1.5 marks)*

(c) X_1 be its first arrival time. Show that given that the number of arrivals in $(0,t]$ is 1, then X_1 is uniformly distributed in $(0,t]$. *(2 marks)*