Quiz 1

Probability and Statistics

Directions

1. Bags needs to be kept either behind the last row or on the stage. No mobiles are allowed.
2. Find nearest seats in front of which answer sheets have been kept. Once seated not allowed to change.
3. Write roll number on all the sheets and the color according to your position before starting. **Red for your right** **|** **Blue for middle** **|** **Green for your left.** Stop writing at 940AM

Any violations will lead to 0 marks for the exam!

1.Suppose 4 balls are randomly assigned to 4 bins (both labelled 1-4). Let A be the event that **1st**|**2nd**|**3rd** ball is sent to the same labelled bin. Let X be number of balls in **3rd**|**4th**|**1st** bin, N be number of non empty bins. Find p_X, p_n, $\mathbf{p_{X,N|A}(x,n)}$|$\mathbf{p_{X|N,A}(x|n)}$|$\mathbf{p_{N|X,A}(n|x)}$ (conditioned on A) for x=1,2,3 and n=2,3. Need to explain your solution. ***(4)***

2. A drunken man at any time takes a step forward with pr. **1/2**|**1/3**|**1/4**, 2 steps backward with pr. **1/4**|**1/6**|**1/8** and stays in place otherwise. He starts at x=0 at time t=0 and goes on till time t=T when he passes out. The time T at which he passes out is 1 with pr. **1/3**|**1/4**|**1/5**, 2 with pr. **1/3**|**1/4**|**1/5** and 3 with probability **1/3**|**1/2**|**3/5**. a.) Find the exp. of the total number of steps (in any direction) he has taken. b.) Let S be the random variable corresponding to his position when he passes out. Find $E[S^5]$. ***(6)***

Question 2

PMF of T $\begin{pmatrix} 1 & 2 & 3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$

PMF of movement $\begin{pmatrix} -2 & 0 & 1 \\ 1/4 & 1/4 & 1/2 \end{pmatrix}$

Let X_1, X_2, X_3 be iid with dist

b.)

Condition on $T = 1, 2, 3$

T=1

$$E[S^5 \mid T=1] = E[X_1^5]$$

$$= \frac{-2^5}{4} + \frac{1}{2}$$

T=2

$$E[S^5 \mid T=2] = E[(X_1+X_2)^5]$$

$$= E\left[X_1^5 + \binom{5}{1} X_1^4 X_2 + \binom{5}{2} X_1^3 X_2^2 + \binom{5}{3} X_1^2 X_2^3 + \binom{5}{4} X_1 X_2^4 + \binom{5}{5} X_2^5\right]$$

$$= E[X_1^5] + \binom{5}{1} E[X_1^4]\,\overset{0}{\cancel{E[X_2]}} + \binom{5}{2} E[X_1^3]E[X_2^2] + \binom{5}{3} E[X_1^2]E[X_2^3] + \binom{5}{4} \overset{0}{\cancel{E[X_1]}}\,E[X_2^4] + \binom{5}{5} E[X_2^5]$$

$$= 2E[X_1^5] + 2 \times \binom{5}{2} E[X_1^2]E[X_2^3]$$

T=3

$$E[S^5 \mid T=3] = E[(X_1+X_2+X_3)^5]$$

Terms:	Expected Value $= 0$	Coefficient
X_i^5		3
$X_i X_j^4$	✓	
$X_i^2 X_j^3$		$\binom{3}{2} \times 2 \times \binom{5}{2}$
$X_i X_j^2 X_k^2$	✓	
$X_i X_j X_k^3$	✓	

Choose i,j from X_1, X_2, X_3

$X_i^2 X_j^3 + X_j^2 X_i^3$

Binomial coeff

$$3E[X_1^5] + \binom{3}{2} \times 2 \times \binom{5}{2} E[X_1^2]E[X_1^3]$$

Question 2

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Let X_1, X_2, X_3 be iid with dist

(a)

Condition on $T = 1, 2, 3$

$T=1$

$\#$ of steps $= \mathbb{E}[|X_1|] = 2 \times \frac{1}{4} + 1 \times \frac{1}{2} = 1$

$T=2$

$\#$ of steps $= \mathbb{E}[|X_1| + |X_2|]$

$= 2\,\mathbb{E}[|X_1|]$ (X_1, X_2 iid, linearity)

$= 2$

$T=3$

$\#$ of steps $= \mathbb{E}[|X_1| + |X_2| + |X_3|]$

$= 3\,\mathbb{E}[|X_1|] = 3$

exp value of total $\#$ of steps

$= \frac{1}{3} \times 1 + \frac{1}{3} \times 2 + \frac{1}{3} \times 3 = 2$